THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and sixty-third regular meeting of the Society was held in New York City on Saturday, April 26, 1913. The attendance at the two sessions included the following fifty-seven members:

Professor R. C. Archibald, Professor M. J. Babb, Dr. F. W. Beal, Mr. R. D. Beetle, Mr. A. A. Bennett, Professor W. J. Berry, Professor E. G. Bill, Professor G. D. Birkhoff, Dr. Henry Blumberg, Professor Maxime Bôcher, Professor B. H. Camp, Professor C. W. Cobb, Dr. Emily Coddington, Professor F. N. Cole, Dr. G. M. Conwell, Professor J. L. Coolidge, Mr. C. H. Currier, Dr. L. L. Dines, Professor L. P. Eisenhart, Professor T. S. Fiske, Professor W. B. Fite, Mr. Meyer Gaba, Mr. G. M. Green, Professor C. C. Grove, Professor H. E. Hawkes, Professor E. V. Huntington, Dr. W. A. Hurwitz, Dr. Dunham Jackson, Mr. S. A. Joffe, Professor Edward Kasner, Dr. S. D. Killam, Mr. E. H. Koch, Jr., Dr. D. D. Leib, Professor W. R. Longley, Professor J. H. Maclagan-Wedderburn, Dr. H. H. Mitchell, Dr. R. L. Moore, Dr. F. M. Morgan, Mrs. Anna J. Pell, Professor A. D. Pitcher, Dr. H. W. Reddick, Professor R. G. D. Richardson, Dr. J. E. Rowe, Professor L. P. Siceloff, Mr. C. G. Simpson, Mr. L. L. Smail, Professor D. E. Smith, Professor P. F. Smith, Professor H. D. Thompson, Mr. H. S. Vandiver, Mr. C. E. Van Orstrand, Professor E. B. Van Vleck, Professor Oswald Veblen, Mr. H. E. Webb, Professor H. S. White, Mr. K. P. Williams, Professor J. W. Young.

The President of the Society, Professor E. B. Van Vleck, occupied the chair, being relieved by Ex-Presidents White and Fiske. The Council announced the election of the following persons to membership in the Society: Professor E. H. Jones, Daniel Baker College; Mr. L. B. Robinson, Johns Hopkins University; Dr. H. M. Sheffer, Cornell University; Dr. W. B. Stone, University of Michigan; Professor F. B. Wiley, Denison University. Six applications for membership in the Society were received.

Professor D. R. Curtiss was appointed a member of the Editorial Committee of the *Transactions*, to succeed Professor

Bôcher, whose term expires in October, 1913. Professor P. F. Smith was appointed to succeed Professor White, who desires to retire from the *Transactions* Committee at the close of the present year.

As already announced, the summer meeting and colloquium of the Society will be held at the University of Wisconsin during the week September 8–13. It was decided to hold the annual meeting this year in New York City on Tuesday and Wednesday, December 30–31.

The following papers were read at the April meeting.

(1) Dr. E. L. DODD: "The error risk of the median compared with that of the arithmetic mean."

(2) Mr. P. M. BATCHELDER: "The divergent series satisfying linear difference equations of the second order."

(3) Mr. P. M. BATCHELDER: "The hypergeometric difference equation."

(4) Mr. H. J. ETTLINGER: "On a generalization of a Sturmian boundary problem."

(5) Dr. R. L. MOORE: "Concerning pseudo-Archimedean and Vollständigkeit axioms."

(6) Dr. J. E. ROWE: "The relation between the pencil of tangents from a point to a rational plane curve and their parameters."

(7) Professor E. G. BILL: "Analytic curves in non-euclidean space (third paper)."

(8) Mr. JOSEPH SLEPIAN: "On the functions of a complex variable defined by a differential equation of the first order and the first degree."

(9) Dr. NATHAN ALTSHILLER: "On the cubic with a double point."

(10) Dr. C. F. CRAIG: "Ruled surfaces associated with certain rational space curves."

(11) Dr. H. M. SHEFFER: "The generalized principle of duality in Boolean algebras."

(12) Dr. T. H. GRONWALL: "On the maximum modulus of an analytic function."

(13) Mr. L. L. SMAIL: "Note on the summability of properly divergent series."

(14) Dr. MAURICE FRÉCHET: "Sur les classes V normales."

(15) Professor MAXIME BÔCHER: "An application of the conception of adjoint systems."

(16) Professor G. D. BIRKHOFF: "Note on the gamma function."

(17) Professor G. D. BIRKHOFF: "Solution of the generalized Riemann problem for linear differential equations, and of the analogous problem for linear difference and q-difference equations."

(18) Professor L. P. EISENHART: "Transformations of surfaces of Guichard."

(19) Professor E. V. HUNTINGTON: "Sets of independent postulates for betweenness (second paper)."

(20) Professor A. D. PITCHER: "On the connection of an abstract set, with applications to the theory of functions of a general variable."

(21) Professor A. D. PITCHER: "Concerning the property Δ of a class of functions."

(22) Professor R. G. D. RICHARDSON: "Oscillation theorems for a system of n linear self-adjoint partial differential equations of the second order with n parameters."

(23) Dr. H. H. MITCHELL: "On some systems of collineation groups."

(24) Mr. H. S. VANDIVER: "Symmetric functions formed by certain systems of elements of a finite algebra, and their connection with Fermat's quotient and Bernoulli's numbers."

(25) Dr. C. A. FISCHER: "The derivative of a function of a surface."

(26) Dr. C. T. SULLIVAN: "Properties of surfaces whose asymptotic curves belong to linear complexes."

(27) Dr. S. D. KILLAM: "A note on graphical integration of functions of a complex variable."

(28) Mr. K. P. WILLIAMS: "On the asymptotic form of the function $\Psi(x)$."

(29) Mr. M. G. GABA: "A set of postulates for general projective geometry in terms of point and transformation."

(30) Dr. W. A. HURWITZ: "Postulate sets for abelian groups and fields."

(31) Professor EDWARD KASNER: "The interpretation of the Appell transformation."

(32) Mr. G. M. GREEN: "Systems of k-spreads in an n-space."

(33) Professor J. W. YOUNG: "A new formulation for general algebra."

(34) Professor J. W. YOUNG and Dr. F. M. MORGAN: "The geometry associated with a certain group of cubic transformations in space."

Mr. Slepian was introduced by Professor Birkhoff, Dr. Altshiller and Dr. Fischer by the Secretary. The papers of Mr. Batchelder and Mr. Ettlinger were communicated to the Society by Professor Birkhoff, and that of Dr. Fréchet by Professor Bôcher. The papers of Dr. Dodd, Mr. Batchelder, Mr. Ettlinger, Dr. Craig, Dr. Sheffer, Dr. Gronwall, Mr. Smail, Dr. Fréchet, Dr. Sullivan, Mr. Williams, Professor Kasner, Mr. Green, Professor Young, and Professor Young and Dr. Morgan were read by title.

The second paper of Professor Pitcher and the paper of Mr. Williams appeared in full in the June BULLETIN. Dr. Killam's paper is published in the present number of the BULLETIN. Dr. Fréchet's paper will appear in the *Transactions*.

Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. It is not at present known what function of the measurements the Gaussian probability law endorses above all other functions; and perhaps there is no such function. The arithmetic mean M of n measurements can not claim this distinction;* indeed, the error risk of $(1 - 1/n^2)M$ is less than that of M when n is large enough.†

Dr. Dodd shows, however, that under the Gaussian law the error risk of the arithmetic mean is less than that of the median.

The median is a function of the measurements which lacks certain partial derivatives at some points, and it can not be given a Taylor development about a point with all coordinates equal. Thus, it is not one of the functions considered by Czuber in his treatment of error risk, "Fehlerrisiko."‡ The median, however, has found considerable favor among biologists and economists; and in certain non-Gaussian distributions is probably superior to the arithmetic mean.

2. The general existence theorem for the theory of linear difference equations, as worked out by Birkhoff, is based on the formal solutions of the equations in terms of divergent series. These series are known to exist only when all the roots of the characteristic equation are distinct from each

* See Bertrand, Calcul des Probabilités (1889), p. 180.

† This BULLETIN, February, 1913, p. 223, inequality (1).

‡ Wahrscheinlichkeitsrechnung, I, p. 276.

other and from zero. The methods of Nörlund and Galbrun, based on the Laplace transformation, lead to great difficulties in the cases of equal or zero roots, while the method of Birkhoff can be applied readily provided a complete set of divergent series has been found. In the present paper Mr. Batchelder proves the existence of formal series for equations of the second order in all possible cases. The difficulties of extending these results to equations of the nth order are chiefly algebraical, and the author hopes shortly to present a complete discussion of the question.

3. In this paper Mr. Batchelder makes a thorough study of the properties of difference equations of the form

$$(a_2x + b_2)f(x + 2) + (a_1x + b_1)f(x + 1) + (a_0x + b_0)f(x) = 0,$$

including all exceptional cases. Assuming first that the characteristic equation possesses two distinct finite roots different from zero, analytic solutions are obtained in the well-known form of definite integrals by means of the Laplace transformtion, and evaluated (with Barnes) in terms of hypergeometric series and gamma functions.

The equation is next shown to admit of formal solution by $S_1(x)$ and $S_2(x)$, two divergent power series in $1/x$ multiplied by exponential factors. Four of the definite integral solutions are characterized by their asymptotic forms as $|x|$ approaches ∞, namely, $g_1(x) \sim S_1(x)$ and $g_2(x) \sim S_2(x)$ in the left half plane, and $h_1(x) \sim S_1(x)$ and $h_2(x) \sim S_2(x)$ in the right half plane; these are the principal solutions. The periodic functions by which $g_1(x)$ and $g_2(x)$ are expressed in terms of $h_1(x)$ and $h_2(x)$ are then determined explicitly, and with their aid the asymptotic forms of the principal solutions are studied in the complete vicinity of ∞. The so-called intermediate solutions are obtained in explicit form, and the properties of the remaining definite integral solutions are studied. The fundamental properties of the solutions of the equation having been thus determined, other questions are considered, such as the conditions for reducibility, etc.

One of the main objects of the paper is reached in the extension of the general theory to the irregular cases in which the roots of the characteristic equation are equal to each other or equal to 0 or ∞. The principal solutions are derived, and the formal solutions which represent them asymptotically for large

values of $|x|$ are obtained in all cases. One interesting fact discovered is that some of the series proceed according to powers of $1/\sqrt{x}$, as in the anormal series for differential equations.

4. In Mr. Ettlinger's paper the following boundary-value problem is considered:

$$\frac{d}{dx}\left[K(x,\lambda)\frac{du}{dx}\right] - G(x,\lambda)u = 0,$$

$$\alpha_0(\lambda)u(a,\lambda) + \beta_0(\lambda)K(a,\lambda)u'(a,\lambda) = \gamma_0(\lambda)u(b,\lambda) + \delta_0(\lambda)K(b,\lambda)u'(b,\lambda),$$

$$\alpha_1(\lambda)u(a,\lambda) + \beta_1(\lambda)K(a,\lambda)u'(a,\lambda) = \gamma_1(\lambda)u(b,\lambda) + \delta_1(\lambda)K(b,\lambda)u'(b,\lambda),$$

$$\alpha_0\beta_1 - \beta_0\alpha_1 = \gamma_0\delta_1 - \delta_0\gamma_1.$$

The existence of an infinite number of characteristic values for this system is proved under the restriction that K and G decrease as λ increases. Mr. Ettlinger makes use of a striking symmetry which is exhibited by the above equation when we set $z = Ku'$, $u' = Hz$, $z' = Gu$, where $H = 1/K$. The method is that used by Bôcher and Birkhoff in treating similar boundary-value problems, and is based on Sturm's fundamental theorems of oscillation. The proof of this theorem rests on the fact that between every pair of characteristic numbers of the associated Sturmian system

$$\alpha_0(\lambda)u(a,\lambda) + \beta_0(\lambda)K(a,\lambda)u'(a,\lambda) = 0,$$

$$\gamma_0(\lambda)u(b,\lambda) + \delta_0(\lambda)K(b,\lambda)u'(b,\lambda) = 0$$

there is to be found at least one characteristic number of the given problem. If $l_1, l_2, \ldots$ be these characteristic numbers and $\lambda_1, \lambda_2, \ldots$ those of the Sturmian system, in general two cases arise, according as $\lambda_{2p-1} \leqq l_{2p} \leqq \lambda_{2p}$ or $\lambda_{2p} \leqq l_{2p} \leqq \lambda_{2p+1}$. Furthermore, if

$$(\alpha_0'\beta_0 - \alpha_0\beta_0') - (\gamma_0'\delta_0 - \gamma_0\delta_0') \geqq 0,$$

$$(\alpha_1'\beta_1 - \alpha_1\beta_1') - (\gamma_1'\delta_1 - \gamma_1\delta_1') \geqq 0,$$

$$(\alpha_0\beta_1' - \alpha_1'\beta_0) - (\gamma_1\delta_0' - \gamma_0'\delta_1) = 0,$$

there is only one value in each such interval. The means for discriminating between these cases is found in the appendix. An oscillation theorem follows for the roots of the solution $u_{p+1}(x)$ which corresponds to $\lambda = l_{p+1}$.

The boundary conditions are then put into a normal form

$$L[u(a, \lambda)] = k(\lambda)L[u(b, \lambda)], \quad M[u(a, \lambda)] = 1/k(\lambda)M[u(b, \lambda)],$$

where

$$L[u(x, \lambda)] = \alpha(\lambda)u(x, \lambda) + \beta(\lambda)K(x, \lambda)u'(x, \lambda),$$

$$M[u(x, \lambda)] = \gamma(\lambda)u(x, \lambda) + \delta(\lambda)K(x, \lambda)u'(x, \lambda),$$

and k may be real or complex with modulus 1. For the case k real, a complete oscillation theorem is obtained for the roots of $L[u_{p+1}(x)]$ and $M[u_{p+1}(x)]$. The special case $k = 1$ requires consideration, and the resulting theorem yields two possibilities for the roots of $M[u_{p+1}(x)]$. For k complex, the oscillation theorem gives no specific information.

5. Consider the following axioms:

Axiom A. If (a) the set of all points of a line l lying in a plane M be divided into two subsets S_1 and S_2 such that no point of S_1 is between two points of S_2; (b) A and B are two points lying in M on the same side of l; (c) the line AB does not separate every point of S_1 (not on AB) from every point of S_2 (not on AB); (d) P_1 is a point of S_1 and P_2 is a point of S_2 and C is a point lying in the plane M on the opposite side of l from A and B; then there exists a point X within the triangle P_1CP_2 such that the triangle AXB contains at least one point of S_1 and at least one point of S_2.

Axiom V. If the set of all points in a plane M be divided into two mutually exclusive subsets S_1 and S_2, then there exists a point X in one of these subsets such that every triangle containing X and lying in M contains points belonging to the other subset.

Let H_1 denote Hilbert's plane axioms of groups I and II,* or Veblen's Axioms I–VIII.† Let H_2 denote H_1 together with Desargues's theorem (considered as an axiom) and, say, Hilbert's Axiom III of parallels.

Dr. Moore shows that

* Grundlagen der Geometrie.

† *Transactions*, vol. 5 (1904), No. 3, pp. 344 and 345.

I. If a plane M satisfies H_2 together with Axiom A, then M is an ordinary euclidean plane with possibly certain points omitted.

II. If a plane M satisfies H_1 and Axiom A, then, though M may not be either a euclidean nor a Lobachevskian plane with respect to the "line" of Hilbert's axioms or of Veblen's definitions, nevertheless there exists in M a set of curves such that if they are taken as the lines of M then M will be an ordinary euclidean plane (with perhaps certain points omitted).

III. If a plane M satisfies H_2, Axiom A, and Axiom V, then it is a categorical euclidean plane.

It seems at least strongly probable however that a plane satisfying H_2 and V is not necessarily a categorical euclidean plane, though a plane satisfying H_2 and "the Dedekind cut postulate" (for every line of that plane) would be a categorical euclidean plane. Thus there seems to be a strong analogy between the Axiom V and Hilbert's axiom of completeness. Clearly Axiom A has some of the properties of an Archimedean axiom.

6. In Dr. Rowe's paper the problem of discovering the relation between the pencil of tangents from a point to a rational plane curve and their parameters is attacked as a purely algebraic problem. The method may be divided into three parts: (1) the derivation of a complete system of covariant curves of the R^n defined by projective relations connecting the pencil of tangents from a point to the R^n, and this requires the further development of a certain type of combinant of two binary forms; (2) the derivation of a complete system of covariant curves of the R^n defined by projective relations connecting the parameters of the tangents from a point to the R^n; (3) a comparison of curves (1) and (2) is carried out in detail for the R^3 and R^4, which reveals the fact that no curve of (1) is expressible in terms of the curves (2) alone, and reasons are cited to support the belief that the same sort of relation exists in the case of rational curves of higher order.

In the case of R^3 we find a line, a cubic, and a sextic each of which possesses the property that from any point of it the pencil of tangents to the R^3 and their parameters are projectively equivalent. In case of R^4 besides the curves from which the pencil of tangents and their parameters are projectively equiva-

lent for a certain set of invariants, we find that from any point of the conic $B = 0$ (in the notation of previous papers) a given invariant relation upon the pencil of tangents to the R^4 is equivalent to some other invariant relation upon their parameters, which relation may easily be found by the method described.

7. This paper is a continuation of the work presented at two meetings of the Chicago Section in which Professor Bill applies to the problems of the differential geometry of non-euclidean space the methods of attack used by Study in discussing similar problems in euclidean space.

8. Mr. Slepian considers the differential equation $dy/dx = P(x, y)/Q(x, y)$, where P and Q are polynomials in the complex variables x and y. The integrals of such an equation are infinitely many valued functions over the x plane, the singularities (branch points and poles) apart from the fixed singularities having a certain uniform distribution. In order that all the fixed singularities of the equation may be of the Briot and Bouquet type, it is necessary that the degree of Q in x be at least two greater than the degree of P in x. With this condition satisfied the following is found to be true:

If the equation admits an algebraic integral, then each integral passes through some fixed singularity of the equation. From this it readily follows that if the degree of P in y does not exceed the degree of Q in y by as much as two, then each integral of the equation passes through some fixed singularity. If each integral of the equation passes through some fixed s ngularity, then each pair of integrals must pass through a common fixed singularity.

For a value x, not the x of a fixed singularity, let an integral $Y(x)$ have in its different branches the values $y_1, y_2, \cdots$; let $z_1, z_2, \cdots$ be limit values of $y_1, y_2, \cdots$; then the integrals which at the point x take the values $z_1, z_2, \cdots$, are called limit solutions of $Y(x)$. If an integral does not pass through a particular fixed singularity, then none of its limit solutions can pass through that fixed singularity.

Any integral of the equation has among its limit solutions either an algebraic integral, or a system of integrals L, having these properties: (1) any limit solution of an integral of the system L is again an integral of the system L; (2) each

integral of the system L is a limit solution of any other integral of the system L. No two different L systems can have an integral in common.

In the neighborhood of a fixed singularity, as is well known, the integrals are given by $f(x, y)^{\lambda}/g(x, y)^{\mu} = C$. If the ratio of λ to μ is not real, then the integrals $f(x, y) = 0$ and $g(x, y) = 0$ are limit solutions of every integral which passes through the fixed singularity. When the ratio of λ to μ is real for none of the fixed singularities of the equation, then there cannot be more than one L system for the differential equation. If, however, for any of the fixed singularities, the ratio of λ to μ is real, then either there are infinitely many L systems or all the integrals of the equation belong to a single L system; in both cases, each integral of the equation passes through some fixed singularity.

If the equation has any integral which passes through no fixed singularity, then there is one and only one L system. Each integral of this system passes through no fixed singularity. The integrals of this system are remarkably like the algebraic functions in their properties.

9. Five given coplanar points A, A', B, B', O are projected from a point M by the involution of rays $M(OO, AA', BB')$. Dr. Altshiller proves synthetically that the locus of M is a cubic with a double point at O, the couples A, A' and B, B' being couples of corresponding poles on the curve. The construction of the curve leads to several interpretations of the conic ω that is enveloped by the lines joining the couples of corresponding poles on the cubic. The tangent to the cubic at any point L, the two tangents from L to ω, and the line LO form an harmonic set. With respect to a point pencil of conics having for its base any two couples of corresponding poles of the cubic, the conjugate of O is on the inflexional line o. Other relations of the points of o to the couples of corresponding poles on the cubic and the conic ω are considered. Incidentally there comes to light a relation between the line pencil of conics determined by a complete quadrilateral and the three point pencils of conics determined by three couples of opposite vertices of the quadrilateral, when taken in pairs.

10. On a rational space curve the lines joining corresponding

points of any projectivity define a rational scroll having the given curve as a component of its nodal curve. In Dr. Craig's paper conditions for coplanar projectivities are determined. The paper includes the rational separability of the scrolls generated by joining a point P to the residual $n-3$ points of intersection of the curve and the osculating plane at P, and a number of other special cases.

11. In this paper Dr. Sheffer proves a principle of duality which holds in Boolean algebras—"the algebra of logic"—when these are determined by means of any set of relations or operations. The well-known duality in terms of $+$, $\times$, 0, 1, and inclusion is shown to be a special case. From the theorem, proved in the paper, that Boolean algebras having more than one element cannot be determined by any postulate set involving self-dual relations alone, it follows that Kempe's "base system" (*Proceedings of the London Mathematical Society*, volume 21, 1890) and Royce's "system Σ" (*Transactions*, volume 6, 1905), supposed by their respective authors to determine Boolean algebras, do so only when the system contains but one element.

12. In this paper, Dr. Gronwall considers the maximum modulus of a power series with given initial coefficients and gives quite elementary proofs of certain theorems obtained by Carathéodory and Fejér (*Palermo Rendiconti*, volume 32, 1911) from considerations belonging to Minkowski's geometry of convex solids.

13. In this note, Mr. Smail gives a very general definition of summability, which includes as special cases the well-known definitions of Cesàro, Riesz, LeRoy, Borel's integral definition, and the definitions of the so-called Cesàro-Riesz methods of Hardy and Chapman, and of Euler's power series method. It is proved in a very simple manner that no properly divergent series can be summable according to this general definition with finite generalized sum, and hence it follows at once that none of the particular methods enumerated above will give a finite generalized sum for such a series.

15. A system consisting of an ordinary homogeneous linear differential equation $L(u)=0$ of the nth order and n homo-

geneous linear boundary conditions $U_i(u) = 0$ at a and b has, as was first pointed out in a general manner by Birkhoff (*Transactions*, volume 9 (1908), page 373), an adjoint system $M(v) = 0$, $V_i(v) = 0$ of the same character, which is such that Green's theorem may be written

$$\int_a^b [vL(u) - uM(v)]dx = U_1V_{2n} + U_2V_{2n-1} + \ldots + U_{2n}V_1,$$

where the U's and V's with subscripts greater than n are expressions of the same sort as those previously introduced with subscripts less than or equal to n. By applications of Green's theorem in this form it may be proved, on the one hand, that the two homogeneous linear systems just mentioned always have the same number of linearly independent solutions; and on the other, that a necessary and sufficient condition that the general linear system

$$L(u) = r, \quad U_1(u) = \gamma_1, \ \ldots, \ U_n(u) = \gamma_n$$

have a solution is that every solution of the adjoint system

$$M(v) = 0, \quad V_1(v) = 0, \ \ldots, \ V_n(v) = 0$$

satisfy the relation

$$\int_a rvdx = \gamma_1V_{2n}(v) + \gamma_2V_{2n-1}(v) + \ldots + \gamma_nV_{n+1}(v).$$

These are the main results of Professor Bôcher's paper. It is shown how the latter includes as a special case a similar result of Mason (*Transactions*, volume 7 (1906), page 337) for the self-adjoint system of the second order; and also how they admit of extension to systems of linear differential equations of any order.

16. Professor Birkhoff's note contains a proof of some well known formulas; it will appear in the BULLETIN.

17. In a paper published in 1909 (*Transactions*, volume 10, pages 436–470), Professor Birkhoff formulated a generalization of the Riemann problem for ordinary linear differential equations, to include the case of irregular singular points; such an extension was first possible after the theorems of this paper.

Later (*Transactions*, volume 12 (1911), pages 242–284) he formulated the problem for linear difference equations which holds, in its field, a position analogous to that of the Riemann problem in the field of linear differential equations.

The solution of these two problems is effected in the present paper by a direct method of successive approximations. The analogous problem for linear q-difference equations is also formulated, and solved by the same means.

18. When a surface S has associated with it a surface $\bar{S}$ such that the two surfaces have the same spherical representation of their lines of curvature, and the principal radii of curvature ρ_1, ρ_2; $\bar{\rho}_1, \bar{\rho}_2$ of the respective surfaces are in the relation $\rho_1\bar{\rho}_2 + \rho_2\bar{\rho}_1 = \text{const.} \neq 0$, then each surface is said to be a surface of Guichard and the other is its associate; these surfaces are named for the distinguished geometer who first considered them. The determination of surfaces of Guichard requires the solution of two partial differential equations, which are of the third order in one dependent variable and of the first order in the other, as Calapso has shown.

Professor Eisenhart has shown that there exist pairs of surfaces of Guichard which constitute the nappes of the envelope of a two-parameter family of spheres on which the lines of curvature correspond. In this sense either surface may be obtained from the other by a transformation of Ribaucour. The determination of the transforms of a surface S requires the solution of an illimitably integrable system of partial differential equations of the first order involving a parameter. When one knows a transformation of S, a transformation of the associate $\bar{S}$ also is known. There is a subclass of surfaces of Guichard, satisfying an additional differential condition involving four arbitrary constants, each of which possesses the property that it admits a transform S_1 of the same kind, involving the same constants, such that the plane determined by the normals to S and S_1 at corresponding points envelopes a surface applicable to a general quadric. Hence the transformations of certain surfaces of Guichard and the deformation of quadrics are allied problems. When S is of the special class referred to above, so also is $\bar{S}$, but the constants are interchanged. With each surface of Guichard there are associated ∞^1 isothermic surfaces. The paper

considers also the transformations of these isothermic surfaces arising from the transformations of the surfaces of Guichard.

19. Professor Huntington's paper is a continuation of the paper presented at the December meeting, and presents six different sets of postulates for betweenness. These six sets are selected from the following list of ten propositions, *A–D*, 1–6, in which the notation *AXB* may be read: "*X* is between *A* and *B*." (*A*) If *AXB*, then *BXA*. (*B*) If *A*, *B*, and *C* are distinct, then at least one of the three relations *ABC*, *BCA*, *CAB* is true. (*C*) If *AXY* and *AYX*, then $X = Y$. (*D*) If *ABC*, then *A*, *B*, and *C* are distinct. In the remaining postulates, 1–6, the elements *A*, *B*, *X*, *Y* are supposed to be distinct. (1) If *XAB* and *ABY*, then *XAY*. (2) If *XAB* and *AYB*, then *XAY*. (3) If *XAB* and *AYB*, then *XYB*. (4) If *AXB* and *AYB*, then either *AXY* or *AYX*. (5) If *XAB* and *YAB*, then either *XYA* or *YXA*. (6) If *XAB* and *YAB*, then either *XYB* or *YXB*. These ten propositions include all the possible formal laws of betweenness concerning not more than four distinct elements. Postulates *A–D* are absolutely independent, and are common to all the sets. Postulates 1–6 contain redundancies, but all the possible ways in which any one of these postulates can be deduced from any other postulates of the list are explicitly set forth in twenty-two theorems. The following six sets are then shown to be the only possible sets of independent postulates that can be selected from the basic list. I. *A–D*, 1, 2. II. *A–D*, 1, 5. III. *A–D*, 1, 6. IV. *A–D*, 2, 4. V. *A–D*, 3, 4, 5. VI. *A–D*, 3, 4, 6. The paper contains twenty-one examples of pseudo-betweenness relations, which are used in the proofs of independence.

20. In this paper Professor Pitcher defines what he calls the connection of an abstract set. By use of this definition he makes a contribution to the theory of functions of a general variable. He secures certain necessary and sufficient conditions which tend to show that the notion connection, as here defined, is fundamental. The paper will appear in the *American Journal of Mathematics*.

22. The chief theorem which Professor Richardson enunciated is general in its character, but was stated for the case of two independent variables and three equations.

Given three two-dimensional regions on the boundary of which the functions $u_1(x_1, y_1)$, $u_2(x_2, y_2)$, $u_3(x_3, y_3)$ shall vanish respectively, and given the equations

$$\frac{\partial}{\partial x_i}\left(p_i\frac{\partial u_i}{\partial x_i}\right)+\frac{\partial}{\partial y_i}\left(p_i\frac{\partial u_i}{\partial y_i}\right)+q_iu_i$$
$$+(\lambda A_{i1}+\mu A_{i2}+\nu A_{i3})u_i=0 \quad (i=1,2,3),$$

where $p_i>0$, q_1, A_{ij} are functions of x_i, y_i; in order that solutions u_1, u_2, u_3 exist which oscillate n_1, n_2, n_3 times respectively, it is necessary that sets of constants $c_1^{(1)}$, $c_1^{(2)}$, $c_1^{(3)}$; $c_2^{(1)}$, $c_2^{(2)}$, $c_2^{(3)}$; $c_3^{(1)}$, $c_3^{(2)}$, $c_3^{(3)}$ exist such that

$$c_1^{(i)}A_{i1}+c_2^{(i)}A_{i2}+c_3^{(i)}A_{i3}=0$$

at least for one value of x_i, y_i, and such that

$$c_1^{(i)}A_{j1}+c_2^{(i)}A_{j2}+c_3^{(i)}A_{j3}\geqq 0,$$

$$c_1^{(i)}A_{k1}+c_2^{(i)}A_{k2}+c_3^{(i)}A_{k3}\geqq 0$$

for all values x_j, y_j and x_k, y_k respectively ($i, j, k = 1, 2, 3$; $i\neq j\neq k\neq i$). These conditions are sufficient for $n_1\geqq n_1'$, $n_2\geqq n_2'$, $n_3\geqq n_3'$ (n_i' is a positive integer or zero; it is certainly zero when $q_i\leqq 0$) if one replaces "at least for one value of x_i, y_i" by "takes both signs."

This paper is a part of an article to appear in the *Mathematische Annalen.*

23. Jordan and Dickson have investigated a system of linear groups in p^m variables, p being any prime, which contain invariant subgroups of order p^{2m+1} or p^{2m+2}. If regarded as a collineation group, any group of this system contains an invariant subgroup of order p^{2m}, and the quotient group is isomorphic with the special abelian linear group on $2m$ indices. From the existence of this system Dr. Mitchell proves the existence, for p an odd prime, of two systems of collineation groups in $(p^m\pm 1)/2$ variables, which are isomorphic with these quotient groups. All three systems have been investigated for $m = 1$ by Klein and Hurwitz and for $m = 2$ by Klein, Witting, and Burkhardt, particularly as regards their relation to elliptic and hyperelliptic functions. The last two do not seem to have been noticed before for $m > 2$.

24. Mr. Vandiver considers symmetric functions formed by a complete system of unit elements of a finite algebra, in particular the finite algebra formed by a complete system of residues of an ideal modulus in an algebraic field; and gives a proof of the theorem

$$\prod_U (x - U) \equiv \sum_{s=1}^{k} \rho_s (x^{\phi(P_s)} - 1)^{\phi(m)/\phi(P_s)} \pmod{m},$$

$$m = P_1^{a_1} P_2^{a_2} \cdots P_k^{a_k}, \quad N(P_s) \neq 2, \quad (s = 1, 2, \cdots, k),$$

the P's being any prime ideals in an algebraic field U. $N(k)$ denotes the norm of k. Furthermore, ρ_s is an integer in the field such that $\rho_s \equiv 1 \pmod{P_s^{a_s}}$ and $\rho_s \equiv 0 \pmod{m/P_s^{a_s}}$. The product extends over a complete system of unit residues modulo m, and x is an indeterminate.

The discussion of functions of the residues of a rational modulus leads naturally to the arithmetical theories of Bernoulli's numbers and Fermat's quotient, and most of the known results relating to these are derived by a uniform process and generalizations are given.

25. In the first part of this paper Dr. Fischer gives a definition of the derivative of a function of a surface analogous to Volterra's definition of the derivative of a function of a curve. If the derivative exists and certain continuity conditions are satisfied, the first variation of the function is proved to be equal to the double integral of the product of this derivative and the variation of the variable z, the integration taking place over that part of the xy-plane over which the given surface is defined. It is also proved that the derivative must vanish identically over a surface which minimizes the value of the given function. In the last part of the paper only those surfaces are considered admissible which give fixed values to a given set of functions. The definition of the derivative of another function of one of these surfaces is modified appropriately and the theorems mentioned above are proved for this restricted set of surfaces also. When the functions considered are defined by means of double integrals the theory developed has an interesting application to the double integral problem of the calculus of variations.

26. In this paper Dr. Sullivan applies the theory of surfaces

as developed by Professor Wilczynski (*Transactions*, 1907 to 1912) to a study of the non-developable surfaces whose asymptotic curves belong to linear complexes.

It is found that: the ruled surfaces have straight line directrices, a result due to A. Peter but established by a method entirely different from that used here. The curved surfaces are the envelopes of either of two one-parameter families of ruled surfaces, having straight line directrices, which belong to complementary reguli of a directrix quadric. They are the integrating surfaces of a canonical system of non-involutory completely integrable partial differential equations of the second order characterized by the following equations for the fundamental invariants:

$$1.\ a' = 0,\ b = UV, \qquad \text{(ruled surfaces)},$$

$$2.\ a' = b = \sqrt{U'V'}/(U + V), \quad \text{(curved surfaces)},$$

where U and V are arbitrary functions of the asymptotic parameters. A geometrical construction is found for surfaces having the property in question.

Special applications are made to certain organic curves and congruences of the surfaces, to the loci of the pinch points of the Cayley cubic scrolls osculating the ruled surfaces, and to the ruled surfaces whose asymptotic curves are twisted cubics.

29. Since geometries are classified according to their characteristic transformations, it would seem that a natural method of postulating a geometry would be in terms of point and transformation. This is done in Mr. Gaba's paper for general projective geometry. The basis is a set of elements called points and a set of transformations of points to points called collineations. Eight postulates upon points and collineations are given, from which are derived as theorems the axioms listed by Veblen and Young in their Projective Geometry.

30. Dr. Hurwitz states a slightly altered form of his set of two postulates for abelian groups (*Annals of Mathematics*, second series, volume 8 (1907), page 94), and with this as basis, constructs a set of five postulates for fields, thus reducing by two postulates the smallest number used hitherto. Consistency and independence are discussed for classes of countably infinite, continuously infinite, and specified finite numbers

of elements. As an incidental result, a simple type of finite commutative non-associative linear algebras is found, admitting unique division by non-zero elements, but not containing idemfacient elements.

31. The Appell transformation, consisting of a projective transformation of the space coordinates together with a certain change in differential of the time, converts any field of force into another field. The trajectories of the two fields are projectively related, but the forces themselves are not so related. Professor Kasner finds the explicit geometric connection of the corresponding forces.

32. According to the general method of Professor Wilczynski, the projective geometry of a given configuration may be studied by means of a completely integrable system of differential equations, of which the integral equations defining the configuration form a fundamental system of solutions. In the present note, Mr. Green points out that in connection with a given completely integrable system of differential equations a number of different geometric configurations may be studied; in particular, he cites the projective geometry of certain systems of k-spreads in an r-spread immersed in an n-space. Thus, in ordinary space, the theories of triple systems of surfaces, of a single system of surfaces, and of a congruence of curves, are all founded on the consideration of the same completely integrable system of differential equations, proper transformations being made in each case.

33. Professor Young's paper is a modification and an extension of the paper presented by him to the Society, December 28, 1911, under the title "On algebras defined by groups of transformations" (see BULLETIN, volume 18 (February, 1912), page 227). In the present paper he considers a system $(\Sigma, \mathfrak{G})$, where Σ is a general class of elements $(a, b, c, \cdots)$ and $\mathfrak{G}$ is a set of transformations on Σ to Σ, with the property that corresponding to every element a of Σ there exists uniquely a transformation G_a of $\mathfrak{G}$. He defines an operation $\circ$ by the relation $G_a(b) = a \circ b$. $\mathfrak{G}_a$ is then represented by the relation $x' = a \circ x$, where x is a variable ranging over Σ; and $\mathfrak{G}$ is the set of transformations $x' = a \circ x$, where the parameter a ranges over Σ. The relations $x' = x \circ a$ constitute the adjoint set $\overline{\mathfrak{G}}$ of

transformations. If $\mathfrak{G}$ is a group, $\overline{\mathfrak{G}}$ is also a group, called the adjoint group. The operation O thus defined between ordered pairs of elements of Σ is very general. It may or may not be one-valued, or associative, or commutative; the inverse operations may or may not exist and, if existent, may or may not be one-valued; there may or may not exist identity elements (i. e., elements i_r or i_l such that $aOi_r = a$ or $i_lOa = a$ for every a); etc. These and similar properties are implied by certain properties of the system $(\Sigma, \mathfrak{G})$, which may be stated readily in the terminology of the theory of transformations. The author lists fourteen such properties and analyses their logical interdependence.

An algebra is obtained by the combination of two or more such elementary systems $(\Sigma, \mathfrak{G})$ which are connected; i. e., by a system $(\Sigma_1, \mathfrak{G}_1; \Sigma_2, \mathfrak{G}_2; \cdots)$ in which $\Sigma_1, \Sigma_2; \cdots$ have common elements. Properties connecting two or more elementary systems of an algebra lead to relations connecting the corresponding operations. Thus necessary and sufficient conditions are obtained in order that one operation be distributive with respect to another, either in the ordinary or in a generalized sense (cf. abstract of earlier paper cited above).

The theory thus developed may readily be applied in various directions. As a very elementary application von Staudt's algebra of points on a line is obtained. By considering operations defined by systems $(\Sigma, \mathfrak{G})$ in which Σ are the points of a projective plane and $\mathfrak{G}$ is a group of collineations, the author shows that the adjoint group may be either a group of collineations, or a group of Cremona transformations of degree n, or a group of transcendental transformations. He applies the results to the determination of all point algebras in a plane having two operations $+$ and $\cdot$ satisfying the associative, commutative, and distributive laws of ordinary algebra, in which the transformations $x' = a + x$ and $x' = a \cdot x$ are collineations. He finds that in the complex plane there are only two essentially distinct algebras of this kind; whereas in the real plane there are three such algebras, one of which is, of course, isomorphic with the algebra of ordinary complex numbers.

34. In a paper presented to the Society, December 28, 1911, entitled "a generalization to 3-space and to n-space of the inversion geometry in a plane" (cf. BULLETIN, volume 18

(1912), page 229), Professor Young called attention to a set of groups $\mathfrak{G}_n$ of Cremona transformations in n-space, which for $n = 2$ reduced to the group of direct circular transforms in a plane. In the present paper Professor Young and Dr. Morgan, after indicating a further generalization of these groups, present a more systematic investigation of the geometry defined by $\mathfrak{G}_3$. This group is defined geometrically as follows: Given a triangle $U_1U_2U_3$ (which for convenience of reference is chosen at infinity) and a projectivity in each of the pencils of planes on the sides of this triangle, a point P (not at ∞) determines uniquely a plane in each of these pencils; if the projectivity transforms these planes into three planes meeting in a point P', P and P' are equivalent under a transformation of $\mathfrak{G}_3$. All the transformations of $\mathfrak{G}_3$ are obtained by varying in all possible ways the projectivities in the pencils of planes referred to.

The "space" of the geometry consists of all "ordinary" points, that is points in the ordinary sense not at ∞; and of all "ideal" points defined as follows: a point U_i ($i = 1, 2, 3$) with any line through U_i, not at ∞, is an ideal point of the first kind; a side of the triangle $U_1U_2U_3$ and any plane through this side is an ideal point of the second kind; the plane at infinity is the ideal point of the third kind. In this space the transformations of $\mathfrak{G}_3$ are one-to-one without exception. The transformations are cubic except when an ideal point of the second kind is invariant, in which case they are quadratic; and when ∞ is invariant, in which case they are linear. The simple one-dimensional elements of the geometry, the so-called characteristic curves, are in general twisted cubics through U_1, U_2, and U_3. For further details see abstract of earlier paper cited above. The simple two-dimensional elements of the geometry, the so-called characteristic surfaces, are: (1) any cubic surface with conical points at U_1, U_2, and U_3 and one free conical point (the "vertex" of the surface). If the vertex of the surface is an ideal point of the first kind at U_i the surface has a binode at U_i; (2) any quadric cone through U_1, U_2, and U_3 with vertex (the "vertex" of the surface) at an ordinary point or on a side of the triangle $U_1U_2U_3$ (in the latter case the "vertex" is an ideal point of the second kind); (3) any plane not on U_1, or U_2, or U_3 (in this case the "vertex" is the point ∞). Among the theorems derived may be mentioned: any two characteristic surfaces are equivalent under

$\mathfrak{G}_3$. The vertex and any other three points (in general position) of a characteristic surface uniquely determine the surface. A characteristic curve through the vertex and two other points of the surface lies completely on the surface.

F. N. COLE,
Secretary.

THE TWENTY-THIRD REGULAR MEETING OF THE SAN FRANCISCO SECTION.

THE twenty-third regular meeting of the San Francisco Section of the Society was held at the University of California, on Saturday, April 12, 1913. The following members of the Society were present:

Mr. B. A. Bernstein, Professor H. F. Blichfeldt, Dr. Thomas Buck, Professor G. C. Edwards, Mr. W. F. Ewing, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. Frank Irwin, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Dr. L. I. Neikirk, Professor E. W. Ponzer, Professor T. M. Putnam.

Professor G. C. Edwards presided at both morning and afternoon sessions. It was decided to hold the next regular meeting of the Section at Stanford University, on October 25, 1913.

The following program was presented:

(1) Mr. B. A. BERNSTEIN: "A set of postulates for the algebra of positive rational numbers with zero."

(2) Professor H. F. BLICHFELDT: "On the arithmetic value of quadratic forms."

(3) Dr. L. I. NEIKIRK: "The analytical geometry of functional space."

(4) Professor T. M. PUTNAM: "Concerning the residues of certain sums of powers of integers to a prime modulus."

(5) Dr. H. W. STAGER: "A geometrical transformation, with some applications to certain systems of spheres."

Abstracts of the papers follow below.

1. In volume 3 of the *Transactions*, Professor Huntington gives a set of postulates for the positive rational numbers. By modifying Professor Huntington's set, Mr. Bernstein obtains a complete set of postulates for the algebra of positive rational numbers with zero.

2. Let f be a positive definite quadratic form in n variables and of determinant D. The following theorem has been proved by Hermite: such a set of integers, not all zero, may be substituted for the variables that the value of f is less than or equal to $\gamma D^{1/n}$, where γ is a number which depends upon n only. Various approximations to γ have been found, that given by Minkowski being the smallest for large values of n. Professor Blichfeldt proves that Minkowski's value for γ may be divided by a number which approaches $\sqrt{2}$ for large values of n.

3. In a paper presented to the International Congress of Mathematicians for 1906 M. Padoa defines all euclidean geometrical concepts in terms of two, the point and the distance, which were left undefined, but subjected to certain postulates. In two papers in the *Nouvelles Annales* M. Fréchet defines the point by an infinite set of numbers $x_1, x_2, x_3, \cdots$ (or for short x) and the distance between the two points $(x, y) = \sqrt{\Sigma(x_j - y_j)^2}$. These satisfy the postulates of M. Padoa and give a generalization of n-dimensional euclidean geometry.

Dr. Neikirk with Kowalewski defines the point by a functional coordinate, $x(j)$, $a \leqq j \leqq b$ (or $0 \leqq j \leqq 1$), and the distance $(x, y)^2 = \int_a^b [x(i) - y(j)]^2 dj$, and by using generalizations of M. Padoa's definitions, gets a generalization of M. Fréchet's geometry.

The n-dimensional plane is defined and shown to be abstractly identical with n-dimensional euclidean space, and the ∞-dimensional plane is also defined and shown to be abstractly identical with Fréchet's space.

The functional space has still higher forms analogous to the plane. The integral equation becomes interpretable as the generalization of the theorem that, in space of n dimensions, n planes of $n - 1$ dimensions intersect in a single point, provided the determinant formed from the coefficients of the coordinates in their equations is not zero.

The paper also considers some other properties of this space. The important results in the paper are the integral relations which are pointed out and the method of their proof suggested by analogy to euclidean geometry.

4. The question of whether or not the congruence $\frac{2^p - 1}{p} \equiv 0 \pmod{p}$ is ever possible, may be made to depend on the

congruence $1^{p-2} + 2^{p-2} + \cdots + \left(\frac{p-1}{2}\right)^{p-2} \equiv 0 \pmod{p}$. Professor Putnam discusses the general congruence of this type, $1^r + 2^r + \cdots + \left(\frac{p-1}{2}\right)^r \equiv a \pmod{p}$, and shows that by expressing a as a fraction it may be given a value for any fixed r (less than p) that is independent of p.

5. In a paper in the *Proceedings of the Edinburgh Mathematical Society*, Professor Allardice considered a geometrical transformation in the plane, $\tan \frac{1}{2}\varphi = k \tan \frac{1}{2}\vartheta$, where ϑ is the angle formed by the enveloping tangents of a curve with a given straight line l, the axis of transformation, and φ is the angle formed by l and t, a system of lines through the intersection of l and t, which envelop the transformed curve of c. In the present paper, Dr. Stager considers analytically a similar transformation in space and applies it to certain systems of spheres. The method of transformation applied to space is as follows: Let α be a given plane and P be any solid. Further, let a plane β be tangent to P and intersect α in i, making with α an angle ϑ. If through i we draw a plane β', making with α an angle φ, such that $\tan \frac{1}{2}\varphi = k \tan \frac{1}{2}\vartheta$, the envelop of β' is defined as the "transform of P." The paper concludes with a number of applications of the method.

W. A. MANNING,
Secretary of the Section.

THE TOTAL VARIATION IN THE ISOPERIMETRIC PROBLEM WITH VARIABLE END POINTS.

BY DR. A. R. CRATHORNE.

(Read before the Chicago Section of the American Mathematical Society, March 22, 1913.)

IN the simple problem of the calculus of variations,

$$J = \int_{x_1}^{x_2} F(x, y, x', y')dt = \text{minimum},$$

the total variation can be expressed as an integral of which the integrand is the Weierstrassian E-function. It is the object of this note to express in a similar way the total vari-

ation for those isoperimetric problems in which the so called Weierstrassian construction is possible.

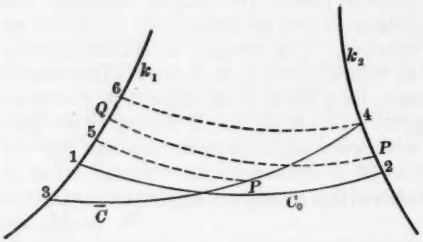
Let k_1: $x = x_1(u)$, $y = y_1(u)$ and k_2: $x = x_2(v)$, $y = y_2(v)$ be two given curves lying wholly or partly in the region R of the problem but not intersecting in this region. The calculus of variations problem under discussion is

$$J = \int_{k_1}^{k_2} F(x, y, x', y')dt = \text{min.}$$

while

$$K = \int_{k_1}^{k_2} G(x, y, x', y')dt = l.$$

Suppose that C_0: $x = x(t)$, $y = y(t)$ is a minimizing curve intersecting the curves k_1 and k_2 in 1 and 2 (figure). C_0



will be one of a two parameter family of extremals which are solutions of the Euler's equation for the problem

$$J = \int H(x, y, x', y')dt = \text{min.}, \quad H = F + \lambda G,$$

and which intersect k_1 transversally. Let the equations of this family be

$$x = \varphi(u, \lambda, t), \quad y = \psi(u, \lambda, t),$$

intersecting k_1 for $t = t_1$. From these we have

$$C_0: \quad x = x(t) = \varphi(u_0, \lambda_0, t), \quad y = y(t) = \psi(u_0, \lambda_0, t).$$

Let

$$\chi(u, \lambda, t) = \int_{k_1}^{t} G(x, y, x', y')dt$$

and consider in the usual way the space curves

$$x = \varphi(u, \lambda, t), \quad y = \psi(u, \lambda, t), \quad z = \chi(u, \lambda, t).$$

Since

$$\chi(u, \lambda, t_1) = 0$$

these curves will all intersect k_1 and they can be shown to fill a portion of space and, with the exception of points on k_1, to form a field R'. Let C_0' be the space corresponding to C_0. Suppose $\overline{C}$: $x = \bar{x}(\tau)$, $y = \bar{y}(\tau)$ to be any other curve connecting k_1 with k_2 for which $K = l$. Let $\overline{C}'$ be the corresponding space curve which, since the Weierstrassian construction is possible, will lie in R'. The equations of $\overline{C}'$ are

$$x = \bar{x}(\tau), \quad y = \bar{y}(\tau),$$

$$z = \bar{z}(\tau) = \int_{\tau_1}^{\tau} G(\bar{x}, \bar{y}, \bar{x}', \bar{y}')dt.$$

Now let the point P (figure) move from 3 to 2, taking the path 34 along $\overline{C}$, then 42 along k_2. First considering P as a point on $\overline{C}$, there will be one extremal $5P$ through P cutting k_1 transversely for which $K_{5P} = \overline{K}_{3P}$. We consider in the usual way* $J_{5P} + \overline{J}_{P4}$, and find

$$J_{\overline{C}} - J_{64} = \int_{\tau_3}^{\tau_4} E d\tau.$$

Following the point P from 4 to 2 along k_2, consider the integral J along the extremals which cut k_1 transversely and for which $K = l$. Let Q be the intersection of these extremals with k_1, then as P moves from 4 to 2, Q will move from 6 to 1. The integral J_{QP} is a function of the parameter v, and using the field integral notation,†

$$J_{QP} = W(x_2(v), y_2(v), z).$$

Since z is constant and equal to l,

$$\frac{dJ_{QP}}{dv} = x_2' W_{x_2} + y_2' W_{y_2} = x_2' H_{x'} + y_2' H_{y'}.$$

The arguments of $H_{x'}$ and $H_{y'}$ are $x_2(v)$, $y_2(v)$, and x' and y'

* Bolza, Variationsrechnung, p. 507.
† Ibid., p. 504.

for the extremal QP at its intersection with k_2. Let $J_{QP} = S(v)$; then

$$J_{64} = S(v_4), \quad J_{c_0} = J_{12} = S(v_2),$$

$$J_{64} - J_{c_0} = -[S(v_2) - S(v_4)] = -\int_{v_4}^{v_2} \frac{dS}{dv}\, dv$$

$$= -\int_{v_4}^{v_2} [x_2' H_{x'} + y_2' H_{y'}] dv,$$

and we have for the total variation

$$\Delta J = J_{\bar{c}} - J_{c_0} = \int_{\tau_3}^{\tau_4} E d\tau - \int_{v_4}^{v_2} [x_2' H_{x'} + y_2' H_{y'}] dv.$$

At the point 2, C_0 cuts k_2 transversally, that is,

$$x_2'(v) H_{x'} + y_2'(v) H_{y'}|^2 = 0,$$

and in this problem, as in the simple problem of the calculus of variations, we are led to a study of the sign of E along the extremal C_0 when considering sufficient conditions.

UNIVERSITY OF ILLINOIS,
March, 1913.

A NOTE ON GRAPHICAL INTEGRATION OF A FUNCTION OF A COMPLEX VARIABLE.

BY DR. S. D. KILLAM.

(Read before the American Mathematical Society, April 26, 1913.)

THE object of this paper is to give a shorter and purely graphical method for graphical integration than that of the author in his thesis* on graphical integration of functions of a complex variable.

We can represent a function $f(z)$ of the complex variable $z = re^{i\theta}$ in the $f(z)$-plane by a system of orthogonal curves $r = r_n$ $(n = 0, 1, \cdots, n)$ and $\theta = \theta_n$ $(n = 0, 1, \cdots, n)$. We choose the values r_n and θ_n so that the $f(z)$ plane is covered by a net of small squares. We seek now a graphical representation in the $Z = X + iY$-plane of the function $Z = \int_0^{z_n} f(z)dz$,

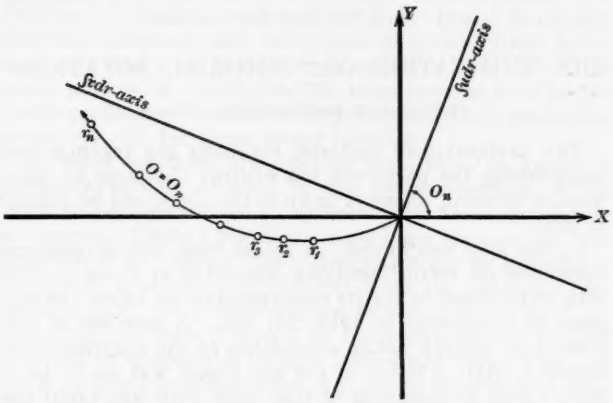
* "Über graphische Integration von Funktionen einer complexen Variabeln mit speziellen Anwendungen," Dissertation, Göttingen, 1912. Referred to in this paper as "thesis."

i. e., we seek the curves $r = r_n$ and $\theta = \theta_n$ in our Z-plane. If we integrate $f(z)$ from $z = 0$ to $z = r_n e^{i\theta_n}$ along the curve or path $\theta = \theta_n$ we have

$$X + iY = Z = \int_0^{r_n} f(z)dz = \int_0^{r_n} \{u(r, \theta_n) + iv(r, \theta_n)\} dr e^{i\theta_n}$$
$$= e^{i\theta_n} \left[\int_0^{r_n} u(r, \theta_n)dr + i \int_0^{r_n} v(r, \theta_n)dr \right],$$

where $f(z) = u(r, \theta) + iv(r, \theta)$.

Now $u(r, \theta_n)$ and $v(r, \theta_n)$ are functions of the real variable r, which we can represent graphically in a $u - r$ and a $v - r$ plane;* and then integrate graphically in order to find the values of $\int_0^{r_n} u dr$ and $\int_0^{r_n} v dr$ $(n = 0, 1, 2, \cdots, n)$.† In our $Z = X + iY$-plane we draw the $\int u dr$ and $\int v dr$ axes so that the angle between the X axis and the $\int u dr$ axis is θ_n. (See figure.)



From equation (1) we see that the factor $e^{i\theta_n}$ means that the X axis must rotate through an angle of θ_n in order to coincide with the $\int u dr$ axis.

In the $\int u dr - \int v dr$ plane we mark the points $R_0, R_1, \cdots, R_n$ with the coordinates $\int_0^{r_n} u dr$ and $\int_0^{r_n} v dr$ $(n = 0, 1, 2, \cdots, n)$.

* See Thesis, p. 15.
† See Thesis, figs. 6 and 7.

These values we get from our graphical integration of the functions u and v, and they can be measured and carried over to the $\int udr - \int vdr$ plane; or by having our $\int udr - \int vdr$ plane on transparent paper, we can mark off the coordinates of the points $R_0, R_1, \cdots, R_n$ without the work of measuring these values. This second method also eliminates a small probable error in measurement.

Through the points $R_0, R_1, \cdots, R_n$ we draw a smooth curve, and this is our required curve $\theta = \theta_n$ in the $\int_0^{z_n} f(z)dz$ plane. In the same way we get the curves $\theta = \theta_0, \cdots, \theta_{n-1}$. Through the points r_n on each curve $\theta = \theta_n$ $(n = 0, 1, \cdots, n)$ we draw a smooth curve, and have a net of small squares covering the Z-plane which is the graphical representation of the function $Z = \int_0^{z_n} f(z)dz$.

UNIVERSITY OF ROCHESTER.

THE UNIFICATION OF VECTORIAL NOTATIONS*

BY PROFESSOR EDWIN BIDWELL WILSON.

THE unification of vectorial notations has taken several steps during the past year, but whether the steps be backward or forward, sideways or up in the air, would be difficult to say.

1. One step was forced. A report from the international committee on vector notations, appointed at Rome in 1908 with instructions to lay its recommendations before the congress at Cambridge in 1912, fell due. A member of that committee, though not in attendance at the congress, I am unable to state whether or not any report was made; but I believe that an extension of time until 1916 was asked and granted. So far as I am aware the committee apparently did not organize prior to the meeting in Cambridge last summer, and except for desultory publication on vectors by a few members of the committee, there had been no inside activity which could lead to a report. It does not appear therefore that much of a step in any direction during the past year or

* This essay may be considered as a continuation of one by the same title in this BULLETIN, May, 1910, p. 415.

the last four years can be attributed to the committee. It may be that something more definite will develop in 1916.

2. In the past four years there has been very great activity in the use of vector methods in Italy. Following the lead of Burali-Forti and Marcolongo, and using their notation, a large number of mathematicians have there printed a great variety of articles on vector analysis and particularly on its applications to geometry and physics. The publishing house of Mattei and Co. in Pavia sent out a circular last summer announcing the proximate appearance of a work on Analyse vectorielle générale consisting of three volumes: 1°. Transformations linéaires (published, 1912); 2°. Applications physico-mécaniques; 3°. Hydrodynamique.

The recent vectorial activity in Italy, and the appearance of such a general work as this, will do much toward fixing the notations of Burali-Forti and Marcolongo in Italian literature (although there are in Italy eminent students of Grassmann's methods, such as A. del Re). It may be recalled that their notations are: italic type for scalars, italic heavy type as $\boldsymbol{a}$ and $\boldsymbol{i}$ for vectors, the large cross as $\boldsymbol{a} \times \boldsymbol{i}$ for the scalar product of two vectors, the large caret as $\boldsymbol{a} \wedge \boldsymbol{i}$ for the vector product, and a great variety of non-algebraic symbolism connected with the linear vector function.

As these notations are different from those in vogue in Germany (system adopted in the German edition of the mathematical encyclopedia), different from those in use here in America (Gibbs's notations must have been pretty well popularized by Coffin's Vector Analysis if not by my edition of Gibbs's lectures), and different from the notations now introduced in the French edition of the mathematical encyclopedia (see below), there is no apparent gain in uniformity of notations attributable to these Italian activities,—although the attendant adoption and use of vectorial methods and ideas has made for a distinct advance of internationalization and uniformity in points of view. Something valuable has thus been accomplished.

3. In the Bulletin of the International Association for Promoting the Study of Quaternions and Allied Systems of Mathematics, June, 1912, Dr. Macfarlane, president of the association, who believes that no satisfactory system of notations can be devised adventitiously, but only, if at all, by a study of fundamental principles, has published a thirty-five

page essay: "A system of notation for vector analysis; with a discussion of the underlying principles." As this essay accomplishes what its title professes, namely, a thorough analysis of those principles which its author, a most distinguished expert in the field, regards as fundamental, and the elaboration of a system of notation upon this analysis as foundation, the work deserves wide circulation among students of vector methods, and conscientious study by them. We hope that the place of publication may not prove a burial ground for the essay.

In the same place is found a tabulation by Shaw, secretary of the association, of the different notations used by different authors. This should be helpful for comparative study; it would have been even more useful had the table been enlarged so as to give in detail as to font, and so on, the fundamental notations of the different systems, even if some abridgment of those parts of the table which deal with derived and less important notations had been necessary. These two contributions by the president and secretary of the organization which, more than any other except the international committee, should be concerned with this problem of unification must occupy an important place in the literature of the problem.

4. To one, however, who believes that the principles which different authors regard as fundamental in vector analysis are even more varied and divergent than the notations actually employed, and who consequently believes that unification of notations can come about, if at all, only by the weight of usage, the most important contribution of the past year to the subject of vector notations appears to be Langevin's article in the French edition of the mathematical encyclopedia.* This is given such high rank in importance because

* Tome IV, vol. 5, fasc. 1, art. IV 16. Notions géométriques fondamentales. Exposé, d'après l'article allemand de M. Abraham (Milan), par P. Langevin (Paris). We may cite also another article appearing last year in the encyclopedia: Tome IV, vol. 2, fasc. 1, art. IV 4. Fondements géométriques de la statique. Exposé, d'après l'article allemand de H. E. Timerding (Brunswick), par Lucien Lévy (Paris). In this second contribution a certain amount of vector analysis is used and the notation, though differing in a detail or two, is in the main the same as Langevin's; it is unfortunate that there are any differences at all, and it almost seems as though French scientists, and the world at large, had a right to demand that uniformity should be enforced at least throughout Tome IV of the Encyclopédie des Sciences Mathématiques. If unification is not attainable on such a small scale, why talk of it at all?

vector analysis, hitherto almost unknown in French literature, has now entered authoritatively into France, because the adoption of any notation in the encyclopedia must have been accompanied by the maturest deliberation (especially in view of the international discussion now going on and of Langevin's membership on the international committee on vector notations), and because, if the notations introduced by him in this article shall be used throughout the appropriate parts of volumes four and five of the encyclopedia, there is likely to be a general decision in their favor throughout France.

In the first place Langevin distinguishes systematically between pure and pseudo-scalars, between polar and axial vectors. Pure scalars and polar vectors are those which do not depend in sign upon any assumption as to positive or negative directions of rotation; whereas pseudo-scalars and axial vectors change their sign when the conventions as to positive and negative direction for rotation are interchanged. The notations

$$a, \quad \underline{a}, \quad \vec{a}, \quad \overset{\curvearrowright}{a}$$

represent respectively pure scalar, pseudo-scalar, polar vector axial vector. That is, a point placed under a letter signifies that the symbol stands for a pseudo-scalar; a straight arrow above a letter, that it stands for a polar vector; a curved arrow, an axial vector.* The font of type has no significance as to the nature of the symbol.

That such a notation, dependent upon the affixing of diacritical marks, has its advantages must be clear. It does not immobilize alphabets or fonts; it may be carried out in blackboard work and in manuscript. Indeed if Clarendons are used for vectors, as is now the nearly universal usage in works on physics, some such notation as a superposed arrow or an underscoring with a dash is necessary for work at the board,—unless one is content to let the physical significance of the symbol, which must of course be always borne in mind, suffice for characterization without any special symbolism. But whether compositors take kindly to these interlinear symbols, which upset the spacing between the lines, is doubtful; and

* It is particularly noteworthy that for the three unit vectors i, j, k no superposed arrow, straight or curved, is used; notationally they therefore rank as scalars.

perhaps the irregular spacing of the lines disfigures the page full as much as the introduction of special fonts which may be justified in.

For a unit vector a zero superscript is suggested; thus $\vec{a}^0$ is a unit vector in the direction of $\vec{a}$. The origin of this notation is implied in the remark "comme a^0 est en algèbre toujours égal à l'unité, nous proposons la notation $\vec{a}^0$ . . . ." Such a reason as this for a notation is not scientific, of course, but merely alliterative; for the basis of the algebraic fact $a^0 = 1$ lies in the equation $a^0 a = a$, which depends on laws not satisfied by the usual products of vectors. It would have been equally proper to remark that since 1 is the arithmetical symbol for unity the notation $\vec{a}^1$ would be adopted for a unit vector along $\vec{a}$. In either case there arises a play upon the word unity, which is naturally somewhat of a convenience as a mnemonic device to correlate a notation and its significance, but which opens the way to confusion if it is considered as anything more profound than a bit of mnemonics. The notation $\vec{a}^0$ has really to be judged solely on the ground of convenience,* and as such it does not seem bad at all.

For the scalar product of two vectors Langevin uses $\vec{a}\ \vec{b}$ or $\vec{a} \cdot \vec{b}$, either juxtaposition or the interposed dot.† As near as I can estimate, the usage is about equally divided in the

* That Langevin considers his notation as based on anything more fundamental than convenience is extremely doubtful; but it is not so with all authors. For instance, there is the commendation which Burali-Forti and Marcolongo bestow on Grassmann for having once adopted (though he subsequently abandoned) the sign $\times$ for the scalar product: "Son chois est très opportune, parce que le signe $\times$ a toute les propriétés que possède le même signe en algèbre dans les produits de deux facteurs." See their Eléments de Calcul vectorielle (traduit par Lattès), p. 222. Now if any one will take the trouble to consult a list of postulates for ordinary algebraic multiplication, for instance Huntington's set (*Annals of Mathematics*,, ser. 2, vol. 2, p. 8), he will find as the first property of the product $a \times b$ that the product is of the same class as the factors (law of closure), and for another property that if $a \times b = 0$, then either $a = 0$ or $b = 0$ (laws of cancellation). Perhaps these are properties not of the sign $\times$, but of the product; if so, what are properties of the sign $\times$ as distinguished from properties of the product, and why?

† Lévy, loc. cit., uses the vertical bar between the letters, $\vec{a} \mid \vec{b}$, as the sign of the scalar product.

text; if the dot had consistently been used, the notation would have been Gibbs's; if it had consistently been omitted, it would have been Heaviside's. Parentheses and brackets have not been immobilized as in the German notation. This seems an advantage.* From some points of view, too, it is an advantage to have the alternative of use or non-use of the dot. Gibbs's adoption of the dot as the sign of the scalar product has always troubled some persons because they wished to be free to use the dot as a separatrix whenever convenient. This difficulty could readily be avoided by the adoption of a small circle, a sort of hollow dot, as the sign of the scalar product, $\vec{a} \cdot \vec{b}$. Prandtl put forth this suggestion in 1904,† and it seems queer that it has not had some vogue.‡

For the sign of the vector product Langevin uses the cross, $\vec{a} \times \vec{b}$, the large ungainly European cross with its arms at sixty degrees instead of ninety. When one follows Gibbs and introduces a cross, it looks best to use a rather small one,§ $\vec{a} \times \vec{b}$; the formulas become more compact and intelligible. It is, however, a matter for congratulation that Langevin has adopted for the scalar and vector products notations which are entirely familiar in the literature instead of devising some totally different affair like our Italian colleagues.

The symbolic operator ∇ is not introduced, but the terms grad,|| div, and rot. There are, further, the monosyllabic symbols Pot, New, Lap, and Max, borrowed from Gibbs, and Lam, invented to represent the inverse of grad. So far as I am aware these integrating operators, proposed by Gibbs (with the exception of Lam), have never found much recog-

* Especially when as in Lorentz's Theory of Electrons (1909) the dot is also always used.

† *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 13, p. 442.

‡ Certainly Gibbs should not be accused of making a fetich of the dot; for when he was discussing with me the question of notation to be adopted in the publication of his Vector Analysis in 1901, he mentioned the disadvantages of immobilizing the separatrix dot and we considered a number of similar symbols as alternatives; if either of us had happened to mention the small circle, it might easily have been adopted instead of the Clarendon dot. Of course the dot may with perfect propriety be considered as a separatrix in Gibbs's system, because the fundamental product is the dyad; and yet it may be undesirable to immobilize the dot in a restricted sense, just as it is undesirable thus to immobilize parentheses.

§ Coffin employed this in his Vector Analysis, and anybody with an eye for typographical beauty must be inclined to follow him.

|| For Langevin grad $f = -\nabla f$, not ∇f. Each author has to choose between the two conventions as to sign, for usage is about equally divided.

nition, and it will be interesting to see if their use by Langevin serves to disseminate them to any appreciable extent. The abandonment of ∇ seems to me unfortunate,* but certain it is that grad, div, and rot are now in almost complete possession of the field and constitute the nearest approach to unification that we have.

Langevin does scarcely more than mention the linear vector function, and he has no particular notation for it; indeed he writes

$$\vec{\rho}' = \vec{\varphi\rho}, \qquad \varphi = i\vec{\alpha} + j\vec{\beta} + k\vec{\gamma},$$

where according to his earlier agreement as to the meaning of juxtaposition φ would reduce merely to a scalar. Convenient as the linear vector function is in many problems, physicists seem little inclined to adopt it, whether in the Hamiltonian, Grassmannian, Gibbsian, or Cayleyan form, and Langevin probably does well not to enter upon it to any extent.

This somewhat full account of the vector notation introduced in the French edition of the encyclopedia seems justified in view of the facts, 1°, that it immobilizes no type, except the cross which is now not much used, and, 2°, that it is adapted to blackboard work without modification. If only a dot or very small circle were used consistently for the sign of the scalar product, the system would be so like Gibbs's that the complete adoption of Gibbs's methods of treating the linear vector function could be adjoined, if and when desired, without any change.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, MASS.,
March, 1913.

* My reasons are fully explained in the reference cited at the beginning of this article.

SHORTER NOTICES.

Grundzüge der Differential- und Integralrechnung. Von Dr. GERHARD KOWALEWSKI, a. o. Professor der Mathematik an der Universität Bonn. Leipzig und Berlin, Teubner, 1909. 452 pp.

IT is the purpose of this book to present a rigorous treatment of the foundations of the calculus. The author therefore begins with a consideration of numbers. Assuming a knowledge of the rational numbers, he defines the irrational number by means of the Dedekind cut, and in the first three chapters, develops the properties of the real number system. In Chapter II (page 14) he defines a limit of a sequence in this way: "Steht eine Folge $u_1, u_2, u_3, \cdots$ zu einer Zahl u in der Beziehung, dass in *jeder* Umgebung von u fast alle Glieder der Folge liegen, so nennt man die Folge konvergent und u ihren Grenzwert." In this definition, he uses the words "fast alle" in the sense of "with a finite number of exceptions." In his notation $\lim u_n = u$ means that there exist u's of the set u_n in every neighborhood of u, with a finite number of exceptions. The equivalence of this definition with the usual form of ϵ statement is established later. Chapter IV deals with functions and variables. y is a function of x when to each x there corresponds a single y. The principal subjects contained in the chapter are the concepts of continuity, the definition and properties of the functions a^x, $\log x$, and finally the derivation of the $\lim_{n=\infty} (1 + 1/n)^n$. The development of this limit is effected with unusual freedom from artifice by means of the inequality

$$(1 + h)^n > 1 + nh \quad (1 + h > 0).$$

The sine and cosine are defined by means of the infinite series, and in preparation for this introduction Chapter VII gives a rather full treatment of the elementary properties of infinite series, concluding with the proof of the theorem on the differentiation of a power series.

Chapters IX and XI are devoted to the theory of maxima and minima of functions of one and two variables, the criteria for a maximum or a minimum for a function of a

single variable being developed on the assumption that the derivative exists at the critical point. Chapter X is on the differentiation of functions of several variables and contains the usual definitions of partial derivatives, total differentials, etc.

For the purpose of introducing the inverse functions the author now proves the theorem that if $f(x)$ takes the same value only once on an interval it is monotonic and so possesses an inverse. Then with the additional hypothesis that $f'(x)$ exists and is not zero on the interval in consideration, he shows that the inverse has a derivative. He is now enabled to complete the differentiation of the elementary functions of trigonometry. In connection with the computation of π he gives an interesting pair of French verses by means of which the value of π may be written down to 30 decimal places. They are as follows:

> Que j'aime à faire apprendre un nombre utile aux sages!
> Immortel Archimède, artiste ingénieur,
> Oui de ton jugement peut priser la valeur!
> Pour moi ton problème eut de pareils avantages.

If in this we replace each word by the number of letters it contains we get as a result the value of π correct to 30 decimals, $\pi = 3.14159265358979323846264338327 9 \ldots$. The remainder of the chapter is devoted to a proof of the theorems on the inverses of systems of functions. Chapter XIII begins the study of integration and comprises the greater and more interesting portion of the book. After proving that every function $f(x)$ continuous on the interval $<a, b>$ (this notation means the end points are included) possesses an integral on this interval, he defines integrability, in general. And for the purpose he uses the following notation: Designate by $\mathfrak{Z}$ any decomposition of the interval $<a, b>$ into any finite number p of sub-intervals. Then any sequence of decompositions $\mathfrak{Z}_1'$, $\mathfrak{Z}_2'$, $\cdots$, $\mathfrak{Z}_n'$ such that $\lim \delta_n = 0$ (where δ_n is the maximal length of any subinterval of $\mathfrak{Z}_n'$) is called a characteristic sequence. Now form the sum $\mathfrak{S}_a^b(z) \equiv (x_1 - a)f(\xi_1) + (x_2 - x_1)f(\xi_2) + \cdots + (b - x_{p-1})f(\xi_p)$ (where ξ_i is any point in the interval (x_i, x_{i-1})). $\mathfrak{S}_a^b(z)$ is called an $\mathfrak{S}$-expression and any sequence of $\mathfrak{S}$-expressions is called a characteristic one if the corresponding $\mathfrak{S}$ sequence is also characteristic. Any function $f(x)$ is integrable in $<a\ b>$ if every characteristic $\mathfrak{S}$-sequence is convergent. It fol-

lows if $f(x)$ is integrable in $< a, b >$ every characteristic $\mathfrak{S}$-sequence has the same limit which he calls $\int_a^b f(x)dx$, and that $f(x)$ is limited in $< a, b >$. Finally $\int_a^b f(x)dx$ exists if, and only if, $f(x)$ is limited in $< a, b >$ and

$$\int_a^{\bar{b}} f(x)dx = \int_{\bar{a}}^b f(x)dx.$$

Chapter XV deals with infinite series. In the consideration of uniform convergence attention is called to the definition given by Goursat in his Cours d'Analyse (Tome I, page 406):

The series of continuous functions

$$u_0(x) + u_1(x) + \cdots + u_n(x) + \cdots$$

convergent in the interval $< a, b >$ is said to be uniformly convergent in this interval if to every positive ϵ there corresponds a positive integer n, independent of x, such that the remainder

$$R_n(x) = u_n(x) + u_{n+1}(x) + \cdots$$

remains in absolute value less than ϵ for all values of x in the interval. This definition is also to be found in Picard (Traité d'Analyse, second edition, page 211) and is that given by Darboux (*Annales Scientifiques de l'Ecole Normale Supérieure*, 1875, 11, series 2, 4, page 77). It requires that $|R_n(x)| < \epsilon$ not for every $n > m$ but only for one value of n. Kowalewski notes that if this definition is employed the theorem on the termwise integrability of a uniformly convergent series is not true. Osgood discussed the subject completely in his review of Goursat's Cours d'Analyse (BULLETIN, 1903, page 552) and calls attention to the fact noticed above by Kowalewski as having been found by Tannery in his Theory of Functions, 1886.

In Chapter XVII there is an extended treatment of the lengths of curves, together with examples of the determination of plane areas.

The treatment of double integrals, differentiation under the sign of integration, curve line integrals, Green's theorem, and the transformations of double integrals are studied quite fully in Chapter XVIII, which concludes the book. There is an appendix dealing with the elementary properties of determinants and concluding with functional determinants.

The typographical errors are neither numerous nor important. The following are perhaps the most noticeable:

Page 57, line 24, should read $\lim f(x_n, y_n) = f(x_0, y_0)$ instead of $\lim f(x_n, y_n) = \lim f(x_0, y_0)$.

Page 85. $\sqrt[n]{u_n}$ instead of $\sqrt[n]{u_n}$.

Page 210, line 9. $\left|\int_a^b f(x)dx - s_a^b(z)\right| \leqq \epsilon$ instead of

$$\left|\int_a^b f(\alpha)dx - s_a^b(z)\right| \leqq \epsilon.$$

Page 213, last line. $\int_a^b f(x)dx$ instead of $\int_a f(x)dx$.

Page 352, line 22. $S' \leqq S$ instead of $S' < S$.

Page 352, line 28. Lim $S(\bar{\mathfrak{Z}})$ instead of lim $S(\mathfrak{Z})$.

The book is admirable for its clearness, conciseness and rigorous style throughout. It has not been the author's design to develop the theorems with a minimum of hypothesis but rather to present them in those forms most usually occurring. There are no problems, but much of the text has been illustrated with well-selected examples.

R. L. BORGER.

Les Fonctions Polyédriques et Modulaires. Par G. VIVANTI, Professeur à la Faculté des Sciences de Pavie. Ouvrage traduit par ARMAND CAHEN, Professeur au Lycée d'Evreux. Paris, Gauthier-Villars, 1910. vi+320 pp.

THE original Italian edition of this useful book appeared in 1906 and was reviewed in this BULLETIN, volume 14 (1908), page 144, by Professor J. I. Hutchinson. No important changes appear in the French translation. Although the work has the modest object of providing an easy introduction to the classic lectures of Klein on the icosahedron and to the treatise on elliptic modular functions by Klein and Fricke, this French translation is evidence of its great usefulness. Unfortunately the number of typographical errors is somewhat large, as was noted by Professor O. Perron in his review published in the *Archiv der Mathematik und Physik*, volume 18 (1911), page 259.

Since the scope and the contents of this work were so well presented in the review by Professor Hutchinson, to which

we referred, we shall simply add that Vivanti did not follow Klein and Fricke in a slavish manner. On the contrary, the work before us gives a masterly independent presentation of some of the most fundamental parts of the theory of linear groups and their geometric interpretation. Many American readers will doubtless welcome this translation into a language with which they are more familiar than that with of the original work.

G. A. MILLER.

Wahrscheinlichkeitsrechnung. Von A. A. MARKOFF. Nach der zweiten Auflage des russischen Werkes übersetzt von H. LIEBMANN. Leipzig and Berlin, B. G. Teubner, 1912. viii+318 pp.

Le Calcul des Probabilités et ses Applications. Par E. CARVALLO. Paris, Gauthier-Villars, 1912. x+170 pp.

The Elements of Statistical Method. By WILFORD I. KING. New York, Macmillan, 1912. xvi+250 pp., price $1.50 net.

THE translation of Markoff's Wahrscheinlichkeitsrechnung will be a welcome companion to that of the same author's Differenzenrechnung. Its motive, as frankly stated in the preface, is to treat the theory simply as a deductive mathematical doctrine, aiming at precision in the analytic formulations, rigor in the proofs, and the determination of superior limits of error where approximate formulas are used. Axioms, definitions, and theorems are formulated explicitly as such, though not always with euclidean austerity of arrangement. The philosophy of the subject is disregarded, and the few special applications admitted receive a relative emphasis determined more by their availability as examples than by their intrinsic interest.

The first four chapters, half of the book, deal with discrete probabilities, elementary and cumulative, their representation by rational numbers, and the classical asymptotic expressions resulting from Stirling's formula. The examples here are entirely from games of chance. Especially noteworthy is the discussion of mathematical expectation.

The rest of the volume contains a short account of the occurrence of irrational numbers as limiting values, together with the notions of probability as applied to continuous sets, with a few of the standard geometric examples; a few pages

on testimony, mortality, and insurance, chiefly as commentary on Bayes' formula and hypothetical probabilities; and a fairly full treatment of the theoretical aspects of the method of least squares. The appendices contain reprints of three of the author's papers, and a compact six-place table of the probability integral.

To a reader whose interest in the subject concerns primarily some of the wide and constantly growing range of applications, the book may appear unnecessarily theoretical and so one-sided. But the choice of material is consistent with its avowed aim, achieved with distinct success, of giving a really logical treatment of the general discipline. It is to be highly commended for furnishing, in concise style and moderate compass, just the kind of treatment needed as antidote to the looseness of thought and expression all too common in statistical literature.

There is a short list of references at the end of each chapter.

Carvallo explains that his object was to provide, for readers of good general education but little technical mathematical training, a modernized and simplified substitute for the classic work of Cournot, which, though still a model of what is possible in the way of a semi-technical treatise, has proved to be not sufficiently accessible to them. The book is directed primarily toward meeting the needs of candidates for the French statistical service, but will hold the attention of any reader interested in the subject, by its vivacity of style, wealth of pungent comment, and the clearness of the verbal explanations which are often given in place of mathematical proofs.

The first chapter discusses the meaning of chance and its laws, the enumeration of elementary events, and the statistical distribution of deviations from a standard; leading to a descriptive treatment of the theorem of Bernoulli by means of numerical examples and the integral curve of error, which is used, no doubt wisely, instead of the more customary exponential frequency curve, its derivative. The second chapter, on statistical method, contains a few pages on mortality formulas and errors of measurement, but is occupied chiefly with a detailed study of the statistics of sex in offspring. The seemingly undue space devoted to this one topic may be explained by the fact of the author's relation to recent statistical inquiries under governmental auspices. This example

is worked through so as to illustrate the various features of the previous theory, as well as the many precautions necessary in sifting statistical data. To the reviewer it seems equally successful as an example of data too hopelessly tangled to leave a chance for faith in any conclusions derived. The third chapter, on the problem of adjustment, treats the method of least squares and the computation of differential corrections. The fourth and last is a collection of loosely related paragraphs on various topics.

The book is distinctly readable, even entertaining, and, in view of its announced purpose, perhaps as successful as could be hoped for. Its faults seem to be chiefly those of omission; for example, it seems to encourage the common but blinding error of supposing that the Bernoulli formula for the distribution of deviations is of universal application. To a reader of reasonable mathematical attainment it will seem like an appetizer rather than a normal meal.

Mr. King's book is intended primarily for students of the social sciences. In general make-up it may be characterized as a kind of condensed and simplified Bowley.

The first two parts, on the history, nature, and uses of statistics and the collection of material, need no comment here, being non-mathematical except for a few hints on arithmetic work. The third part, on the analysis of material, makes enough use of elementary mathematical concepts to be noticed briefly.

The three chapters on tables, diagrams, and graphs are satisfactory so far as they go, except for some peculiar remarks on the smoothing of curves. Of the remainder of the book, dealing with types and averages, dispersion, skewness, variation, and correlation, it may be said that it is no worse and no better than the usual standard of books of its class, and is to be criticized chiefly for certain objectionable features which are unfortunately rather common, and are especially misleading to students without experience in such work. The unfounded sweeping generalizations, the use of words in a technical way with no indication of their meaning, the show of mathematical precision in the treatment of concepts whose inherent vagueness makes it utterly fictitious, are strangely out of place in a book claiming to be a guide to scientific study.

A. C. Lunn.

NOTES.

THE concluding (June) number of volume 14 of the *Annals of Mathematics* contains the following papers: "On the uniformization of algebraic functions" by W. F. OSGOOD; "Manifolds of n dimensions," by O. VEBLEN and J. W. ALEXANDER; "Systems of plane curves whose intrinsic equations are analogous to the intrinsic equation of an isothermal system," by H. W. REDDICK; "The probability of the arithmetic mean compared with that of certain other functions of the measurements," by E. L. DODD; "Cusp and undulation invariants of rational curves," by J. E. ROWE.

THE Paris academy of sciences has awarded the biennial Petit d'Ormay prize of 10000 francs to Professor C. GUICHARD, of the University of Paris, for his contributions to mathematics.

THE Belgian academy of sciences announces the following prize problem for the year 1914:

"Present a contribution to the study of the properties of analytic functions which cannot take certain values in a given domain."

Competing memoirs should be prepared under the usual conditions and be in the hands of the secretary before August 1, 1914. The prize is fr. 1000.

AN international committee has recently been formed to promote the establishment of uniform notations in the theories of potential and elasticity by international cooperation. In its first circular the committee requests from astronomers, mathematicians, and physicists replies to the following question: What are the notions and notations in respect of which it is desirable to establish uniformity? The replies will be collated as soon as possible, and a second circular will then be issued inviting suggestions as to the methods by which uniformity may be brought about. The subject will then be laid before the International congress of mathematicians in 1916 for discussion, and again in 1920 for a final decision.

Replies and inquiries should be addressed to Professor ARTHUR KORN, Schlüterstrasse 25, Charlottenburg-Berlin, Germany.

THE annual meeting of the French association for the advancement of science was held at Tunis, during the week from March 22 to 29. The section of mathematics was under the presidency of M. MOURGNOT, chief of the topographical service of Tunis. Twenty-eight papers were read before the section, as follows: (1) "Tables of successive prime numbers of eight and of nine figures," by A. Gérardin; (2) "Mathematical game," by A. Gérardin; (3) "A calendar discovered by H. Tarry," by A. Gérardin; (4) "Life of A. Gauthier," by E. Lebon; (5) "Life of H. Poincaré," by M. Mourgnot; (6) "A problem of enumeration," by C. Halphen; (7) "Equality of the nth degree," by G. Tarry; (8) "Organization of instruction in Tunis," by M. Mourgnot; (9) "Practical means of finding the day of the week of a given date," by M. Cuénod; (10) "Some new properties of inscribed quadrilaterals," by M. Durel; (11) "Criticism of Darwin's hypothesis of the origin of the moon," by E. Belot; (12) "A new criterion for determining whether a given number is prime," by E. N. Barisien; (13) "On two ellipses derived from the circle of Joachimsthal," by E. N. Barisien; (14) "Extension of the limaçon of Pascal," by E. N. Barisien; (15) "Application of Volterra's equation to certain problems in life insurance," by R. Risser; (16) "Comparison of the Julian or Gregorian with the Mohammedan calendar," by M. Gardès; (17) "Construction of the center of curvature of the ellipse," by M. Balitrand; (18) "A theorem on the involute of the ellipse," by M. Balitrand; (19) "Foucault's pendulum," by C. Litre; (20) "Mathematical instruction," by J. Richard; (21) "A system of measurement," by M. Kesselmeyer; (22) "Establishment of a provisional mortality table," by R. Risser; (23) "The arithmetician Frenicle," by A. Aubry; (24) "On nomography," by F. Boulad; (25), (26), (27), three communications by Dr. Welsch, titles not given; (28) "Recent work in astronomy," by M. Montangerand.

The next meeting of the association will be held at Havre, in August, 1914. The president of the section of mathematics is Dr. REBIÈRE, of Havre, and the secretary is Dr. A. GÉRARDIN, of Nancy.

THE following courses in mathematics are announced for the academic year 1913–1914:

JOHNS HOPKINS UNIVERSITY.—By Professor F. MORLEY:

Higher geometry, three hours (first half year); Theory of functions, three hours (second half year).—By Professor A. B. COBLE: Discontinuous groups, two hours.—By Dr. A. COHEN: Differential geometry, two hours; Theory of functions, two hours.—By Mr. H. P. BATEMAN: Theory of the potential, one hour.

PRINCETON UNIVERSITY.—By Professor H. B. FINE: Theory of functions of a complex variable, three hours.—By Professor H. D. THOMPSON: Infinitesimal geometry, three hours.—By Professor L. P. EISENHART: Projective geometry, three hours; Seminar, two hours.—By Professor P. BOUTROUX: Differential equations, three hours.—By Professor E. P. ADAMS: Electricity and magnetism, three hours; Analytic mechanics, three hours.—By Professor E. SWIFT: Theory of the Newtonian potential, three hours.—By Professor H. McL. WEDDERBURN: Higher analysis, three hours.—By Dr. T. H. GRONWALL: Analytic theory of numbers, three hours.

LORD RAYLEIGH, chancellor of the University of Cambridge, received the honorary degree of doctor of civil law at the recent convocation exercises of Durham University.

PROFESSOR E. PICARD, of the University of Paris, has been elected to membership in the royal Hungarian academy of sciences of Budapest.

PROFESSOR F. HAUSDORFF, of the University of Bonn, has accepted the professorship of mathematics at the University of Greifswald.

DR. E. JACOBSTHAL has been appointed docent in mathematics at the technical school at Berlin.

PROFESSOR I. SCHUR, of the University of Berlin, has been appointed professor of mathematics at the University of Bonn.

PROFESSORS G. D. BIRKHOFF, J. L. COOLIDGE, and E. V. HUNTINGTON, of Harvard University, have been elected to membership in the American academy of arts and sciences.

PROFESSOR S. EPSTEEN, of the University of Colorado, has been appointed State Commissioner of Insurance and has received academic leave of absence for the coming year.

PROFESSOR J. W. GLOVER, of the University of Michigan, has been appointed expert special agent of the bureau of the United States census to supervise the preparation of a special volume on vital statistics. Extensive and systematic mortality tables are to be prepared on the population and the vital statistics of the United States.

DR. S. E. BRASEFIELD has been appointed assistant professor of mathematics at Rutgers College.

DR. S. D. KILLAM has been appointed instructor in mathematics in the University of Alberta.

DR. H. W. CRAMBLET has been appointed instructor in mathematics at the University of Rochester.

PROFESSOR H. WEBER, of the University of Strassburg, died May 17, at the age of 71 years.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

APPELL (P.). See PAPELIER (G.).

BERGHOLZ (O. A.). Substitutionsbeweis des grossen Fermatschen Satzes auf Grund der Formel für $(a+b)^2$. Dussau, Art'l, 1913. 8vo. 22 pp. M. 1.50

BLASCHKE (W.). See STUDY (E.).

CAPRILLI (A.). Nuove formole d'integrazione. Livorno, Belforte, 1912. 16mo. 7 + 178 pp. L. 4.00

CHARLES (R. L.). See FRANKLIN (W. S.).

COTTY (G.). Les fonctions abéliennes et la théorie des nombres. (Thèse.) Toulouse, Privat, 1912. 8vo. 173 pp.

CREFCOEUR (A. J. M.). Cours d'analyse infinitésimale. 1re partie: Calcul différentiel et calcul intégral. 2e édition, revue et augmentée. Paris, Béranger, 1912. 358 pp. Fr. 8.00

DIENES (P.). Leçons sur les singularités des fonctions analytiques. Paris, Gauthier-Villars, 1913. 8vo. 8+172 pp. Fr. 5.50

DUPLESSY (A.). Esquisse de morale mathématique pour parvenir au calme intime. Dijon, Coquart, 1912. 18mo. 48 pp.

ENRIQUES (F.). Les concepts fondamentaux de la science. Leur signification réelle et leur acquisition psychologique. Traduit par L. Rougier. Paris, Flammarion, 1913. 18mo. 315 pp. Fr. 3.50

FINGER (C.). Einteilung der rationalen Raumkurven vierter Ordnung. (Diss.) Münster, 1912. 8vo. 27 pp.

FLEISCHHAUER (H.). Allgemeiner Beweis des grossen Fermatschen Satzes. Delitzsch, Pabst, 1913. 8vo. 8 pp. M. 0.75

FRANKLIN (W. S.), MACNUTT (B.) and CHARLES (R. L.). Elementary treatise on calculus; a textbook for colleges and technical schools. South Bethlehem, Pa., 1913. 8vo. 10 + 253 + 41 pp. Cloth.

GEIGER (W.). Ueber das logarithmische Potential einer gewissen Ovalfläche. (Diss.) Halle, 1912. 4to. 53 pp.

HEIBERG (J. L.). See TANNERY (P.).

JORDAN (C.). Cours d'analyse de l'Ecole polytechnique. 3e édition, revue et corrigée. Tome 2: Calcul intégral. Paris, Gauthier-Villars, 1913. 8vo. 712 pp. Fr. 20.00

KILLAM (S. D.). Ueber graphische Integration von Funktionen einer komplexen Variabeln mit speziellen Anwendungen. (Diss.) Göttingen, 1912. 8vo. 31 pp.

KNOPP (K.). Funktionentheorie. 1ter Teil: Grundlagen der allgemeinen Theorie der analytischen Funktionen. (Sammlung Göschen, Nr. 668.) Berlin, Göschen, 1913. 8vo. 142 pp. Cloth. M. 0.90

KOENIGS (G.). See SAINTE-LAGUË (A.).

KOZMINSKY (I.). Numbers, their meaning and magic. Boston, Occult and Modern Thought Book Center, 1913. 12mo. 5+100 pp. Paper. $0.50

LÖFLUND (F.). Ueber algebraische Raumkurven. (Diss.) Tübingen, 1912. 8vo. 44 pp.

LÖWENBERG (E.). Der grosse Fermatsche Satz. Ur- und Grundbeweis für seine Unmöglichkeit. Berlin, Schildberger, 1913. 8vo. 4 pp. M. 0.50

LÜCKHOFF (W.). Allgemeiner Beweis des Fermatschen Satzes. Berlin-Wilmersdorf, Lückoff, 1913. 8vo. 4 pp. M. 0.50

MACNUTT (B.). See FRANKLIN (W. S.).

MCGINNIS (M. A.). Proof of Fermat's theorem, and McGinnis' theorem of derivative equations in an absolute proof of Fermat's theorem; reduction of the general equation of the fifth degree to an equation of the fourth degree. Meadowdale, Wash., Hjorth, 1913. 12mo. 11+34 pp. $2.00

MORIN (H. de). Les appareils d'intégration; intégrateurs simples, planimètres, intégromètres, intégraphes et courbes intégrales, analyseurs harmoniques. (Bibliotèque Générale des Sciences.) Paris, Gauthier-Villars, 1913. 8vo. 4+208 pp. Fr. 5.00

NOAILLON (P.). Développements asymptotiques dans les équations différentielles linéaires à paramètre variable. Bruxelles, Hayez, 1912. 200 pp. Fr. 3.50

OETTINGEN (A. J. v.). See STEINER (J.).

PAPELIER (G.). Leçons sur les coordonnées tangentielles. Avec une préface de P. Appell. 1re partie: Géométrie plane. 2e édition. Paris, Vuibert, 1913. 8vo. 6+331 pp.

PAPERS set in the mathematical tripos, part 1, in the University of Cambridge, 1908–1912. Cambridge, University Press, 1913. 8vo. 2s. 6d.

PAWLOWSKI (G. de). Voyage au pays de la quatrième dimension. Paris, Fasquelle, 1912. 18mo. 328 pp. Fr. 3.50

PLATRIER (C.). Sur les mineurs de la fonction déterminante de Fredholm et sur les systèmes d'équations intégrales linéaires. (Thèse.) Paris, Gauthier-Villars, 1913. 4to. 77 pp.

POLETTI (L.). Un contributo alla tavola dei numeri primi. Pontremoli, Cavanna, 1913. 8vo. 9 pp.

ROUGIER (L.). See ENRIQUES (F.).

SAINTE-LAGUË (A.). Notions de mathématiques, avec préface de G. Koenigs. Paris, Hermann, 1913. 8vo. 7+512 pp. Fr. 7.00

SALPETER (J.) Einführung in die höhere Mathematik für Naturforscher und Ärzte. Jena, Fischer, 1913. 8vo. 13+336 pp. M. 13.00

SCHRÖDER (G.). Der grosse Fermatsche Satz. Ein mathematisches Problem gelöst. Berlin, Simion, 1913. 8vo. 5+63 pp. M. 4.00

SILVERMAN (L. L.). On the definition of the sum of a divergent series. (The University of Missouri Studies. Mathematics Series, volume 1, number 1.) Columbia, Mo., University of Missouri, 1913. 8vo. 100 pp. $1.00

STEINER (J.). Die geometrischen Constructionen ausgeführt mittelst der geraden Linie und eines festen Kreises. (Ostwald's Klassiker. Neue Auflage. Nr. 60.) Herausgegeben von A. J. v. Oettingen. 2te Auflage. Leipzig, Engelmann, 1913. 8vo. 85 pp. M. 1.50

STUDY (E.). Vorlesungen über ausgewählte Gegenstände der Geometrie. 2tes Heft: Konforme Abbildung einfacher zusammenhängender Bereiche. Herausgegeben unter Mitwirkung von W. Blaschke. Leipzig, Teubner, 1913. 8vo. 5+142 pp. M. 5.60

TANNERY (P.). Mémoires scientifiques. Publiés par J. L. Heiberg et H. G. Zeuthen. Tome II: Sciences exactes dans l'antiquité (1883–1898). Paris, Gauthier-Villars, 1913. 4to. 12+555 pp.

THUE (A.). Ueber eine Eigenschaft, die keine transcendente Grösse haben kann. (Videnskapsselskapets Skrifter, I, Nr. 20.) Kristiania, Dybwad, 1912. 8vo. 15 pp. M. 1.00

——. Ueber die gegenseitige Lage gleicher Teile gewisser Zeichenreihen. (Videnskapsselskapets Skrifter, I, Nr. 1.) Kristiania, Dybwad, 1912. 8vo. 67 pp. M. 2.25

VOIGT (A.). Untersuchung der Gleichung $x^m - y^m = z^m$. Frankfurt, Adelmann, 1913. 8vo. 12 pp. M. 0.50

WEITZEL (C. G.). Unterrichtsbriefe zur Einführung in die höhere Mathematik, in Gesprächsform zum Selbstunterrichte und für solche, die beim Erlernen der Mathematik Schwierigkeiten haben. In 30 Lieferungen. 1te Lieferung. Wien, Hartleben, 1913. 8vo. 8+32 pp. M. 0.50

ZEUTHEN (H. G.). See TANNERY (P.).

II. ELEMENTARY MATHEMATICS.

AMIOT (A.). Trattato di geometria elementare. Nuova edizione, di A. Socci. Parte II: geometria solida. 7a impressione. Firenze, Le Monnier, 1912. 8vo. 221 pp. L. 2.00

AMODEO (F.). Aritmetica particolare e generale, ed algebra. 3a edizione. Napoli, Pierro, 1912. 16mo. 22+241 pp. L. 1.50

——. Complementi di analisi algebrica elementare, con appendice sulle sezioni coniche. 2a edizione, riveduta e migliorata. Napoli, Pierro, 1912. 16mo. 34+229 pp. L. 3.00

ARZELÀ (C.). Trattato di algebra elementare. 3a edizione, 6a impressione. Firenze, Le Monnier, 1912. 16mo. 11+503 pp. L. 3.50

AUBERT (P.) et PAPELIER (G.). Exercices d'algèbre, d'analyse et de trigonométrie. Tome 1er. 2e édition. Paris, Vuibert, 1913. 8vo. 366 pp.

BALLERINI (P.). Elementi di algebra. 4a edizione. Monza, Artigianelli, 1912. 16mo. 252 pp. L. 2.00

——. La geometria. 3a edizione. Monza, Artigianelli, 1912. 16mo. 269 pp. L. 2.50

——. La trigonometria piana. Monza, Artigianelli, 1912. 16mo. 145 pp. L. 2.00

BAUER (W.) und HANXLEDEN (E. v.). Planimetrie nebst Stereometrie und Trigonometrie. Braunschweig, Vieweg, 1913. 8vo. 8+252 pp. Cloth. M. 4.00

BERTRAND (G.). Trattato di algebra elementare. Nuova edizione, di A. Socci. Firenze, Le Monnier, 1912. 16mo. 4+292 pp. L. 2.00

BIÉMONT (P.). See CÉSAR (J.).

BOURLET (C.). Cours de mathématiques. Eléments d'analyse et de géométrie analytique. 2e édition, entièrement refondue. Paris, Gauthier-Villars, 1913. 8vo. 6+252 pp. Fr. 8.00

BRUNO (G. M.). Primeras nociones de algebra, con numerosos ejercicios. Madrid, 1910. 16mo. 92 pp.

CAMMAN (P.) et RÉBOUIS (A. G.). Cours élémentaire de géométrie plane. 2e édition revue et corrigée. Paris, Gigord, 1913. 12mo. 307 pp. Fr. 1.75

CASTLE (L. G.). Mathematics, science and drawing for the preliminary technical course. London, Routledge, 1913. 8vo. 158 pp. 1s.

CÉSAR (J.) et BIÉMONT (P.). Traité d'arithmétique financière. Paris, Hermann, 1912. 8vo. 170+36 pp. Fr. 4.25

CRANTZ (P.). Lehrbuch der Mathematik. 2ter Teil: Für Oberlyzeen. 2te Auflage. Leipzig, Teubner, 1913. 8vo. 5+221 pp. M. 2.40

DE GROODT (G.). Cours de trigonométrie rectiligne théorique et pratique. 2e édition. Namur, Lambert, 1912. 109 pp. Fr. 1.50

FITE (W. B.). College algebra. Boston, Heath, 1913. 8vo. 6 + 283 pp. Cloth.

FORTIN (C.). La géométrie des baccalauréats littéraires. 1re partie. Paris, Société Saint-Augustin, 1913. 8vo. 83 pp.

GODFREY (C.) and SIDDONS (A. W.). Elementary algebra. Volume 2, with answers. Cambridge, University Press, 1913. 8vo. 362 pp. 2s. 6d.

——— — ———. Four-figure tables. Cambridge, University Press, 1913. 8vo. 40 pp. 9d.

GRÉVY (A.). Géométrie plane. 5e édition. Paris, Vuibert, 1913. 8vo. 296 pp. Fr. 3.00

HACCOUR (M.). Cours élémentaire d'algèbre théorique et pratique. 4e édition. Namur, Wesmael-Charlier, 1912. 256 pp. Cartonné. Fr. 2.00

HALL (H. S.). Examples in algebra, taken from Part I of School Algebra. With answers. London, Macmillan, 1913. 8vo. 8+168+37 pp. 2s.

HANXLEDEN (E. v.). See BAUER (W.).

JORDAN (W.). Opus palatinum. Sinus- und Cosinus-Tafeln von 10″ zu 10″. 2te, berichtigte Auflage. Hannover, Hahn, 1913. 8vo. 7+270 pp. M. 7.00

KLEIN (F.). See SCHRÖDER (J.).

LANDE (G. De La). Tavole di logaritimi, estese a sette decimali da F. G. Marie. Napoli, Morano, 1913. 16mo. 12+208 pp. L. 1.50

LESSER (O.). See SCHWAB (K.).

LOW (D. A.). Practical geometry and graphics. New York, Longmans, 1913. 8vo. 8+448 pp. $2.50

MACÉ DE LÉPINAY (A.). Compléments d'algèbre et notions de géométrie analytique. 6e édition. Paris, Vuibert, 1913. 8vo. 486 pp.

MANNHEIMER (N.). See REINHARDT (W.).

MOUGIN (E.). Nouvelles tables de logarithmes. Roanne, Loire, E. Mougin, 1913. Fr. 1.50

OLIVEIRA (J. X. de). Goniometria do tiro indirecto, com o curso de estado maior e engenharia. 2° edição, correcta e augmentada. Paris, Briguet, 1913. 18mo. 13+160 pp.

OTT (K.). Die angewandte Mathematik an den deutschen mittleren Fachschulen der Maschinenindustrie. (Abhandlungen, Band IV, Heft 2.) Leipzig, Teubner, 1913. 8vo. 6+128 pp. M. 4.00

PAPELIER (G.). See AUBERT (P.).

PERRY (J.). Elementary practical mathematics. London, Macmillan, 1913. 8vo. 350 pp. 6s.

POUTHIER (E.). Pour qu'on apprenne les mathématiques. Préface de B. Baillaud. Paris, Didier, 1912. 16mo. 14+407 pp. Fr. 3.50

RÉBOUIS (A. G.). See CAMMAN (P.).

REINHARDT (W.), MANNHEIMER (N.) und ZEISBERG (M.). Geometrie. Frankfurt, Diesterweg, 1913. 8vo. 1ter Teil: 7+142 pp. 2ter Teil: 6+138 pp. Cloth. M. 4.20

SALOMON (A.). Leçons de géométrie. 7e édition. Paris, Vuibert, 1912. 16mo. 6+258 pp. Fr. 2.00

SCHMEHL (C.). Die Algebra. Anhang: Die Anfangsgründe der Differentialrechnung. Giessen, Roth, 1913. 8vo. 4+22 pp. M. 0.50

———. Aufgaben aus der analytischen Geometrie der Ebene. 2te, verbesserte Auflage. Giessen, Roth, 1913. 8vo. 7+111 pp. M. 2.00

SCHRÖDER (J.). Die neuzeitliche Entwicklung des mathematischen Unterrichts an den höheren Mädchenschulen Deutschlands. (Abhandlungen, Band I, Heft 5.) Mit einem Schlusswort zu Band I von F. Klein. Leipzig, Teubner, 1913. 8vo. 12+3+183 pp. M. 6.00

SCHULZ (P.). Fünfstellige logarithmische und trigonometrische Tafeln. Nürnberg, Koch, 1913. 8vo. 112 pp. M. 1.40

SCHWAB (K.) und LESSER (O.). Geometrie und Trigonometrie. 3ter Teil: Lehrstoff der Klassen II und I. Leipzig, Freytag, 1913. 8vo. 229 pp. Cloth. M. 3.00

SIBIRANI (F.). Elementi di algebra. 3a edizione. Firenze, Le Monnier, 1913. 16mo. 145 pp. L. 1.50

SIDDONS (A. W.). See GODFREY (C.).

SOCCI (A.). Nozioni di geometria elementare. Parte I. Firenze, Le Monnier, 1912. 8vo. 115 pp. L. 1.00

—— e TOLOMEI (G.). Elementi di algebra. Nuova edizione, completamente rifatta. Firenze, Le Monnier, 1912. 8vo. 8+281 pp. L. 3.00

——. See AMIOT (A.) and BERTRAND (G.).

SPEZIA (L.). Tavola dei valori della circonferenza e della superficie di circoli, coi rispettivi logaritmi. Torino, Tibaldi, 1912. 8vo. 24 pp.

TABLES de logarithmes à cinq décimales. Par F. I. C. Edition simple. Division sexagésimale. Paris, Gigord, 1913. 12mo. 8+148 pp.

TESTI (G. M.). Corso di matematiche. Volume 3: Geometria elementare. 3a edizione, corretta e modificata. Livorno, Giusti, 1913. 8vo. 10+411 pp. L. 3.50

TOLOMEI (G.). Tavole di logaritmi con cinque decimali. 5a impressione. Firenze, Le Monnier, 1913. 16mo. 200 pp. L. 1.50

——. See SOCCI (A.).

VIELER (A.). Raumlehre. 3te Auflage. Leipzig, Teubner, 1913. 8vo. 8+190 pp. Cloth. M. 2.00

VUIBERT (H.). Problèmes de baccalauréat (mathématiques). 6e édition. Paris, Vuibert, 1913. 8vo. 531 pp.

ZEISBERG (M.). See REINHARDT (W.).

III. APPLIED MATHEMATICS.

BAKER (J. O.). Principes du nivellement géodésique. Traduit et annoté par L. Pennequin. Paris, Terquem, 1912. 18mo. 154 pp. Fr. 3.50

BARBILLION (L.). Production et emploi des courants alternatifs. (Collection Scientia.) 2e édition, entièrement refondue. Paris, Gauthier-Villars, 1912. 8vo. 100 pp. Cartonné. Fr. 2.00

BJERKNES (V.). Die Meteorologie als exakte Wissenschaft. (Antrittsvorlesung.) Braunschweig, Vieweg, 1913. 8vo. 16 pp. M. 0.80

——. Dynamische Meteorologie und Hydrographie. Autorisierte deutsche Uebersetzung von F. Kirchner. 2ter Teil: Kinematik der Atmosphäre und der Hydrosphäre. Von V. Bjerknes, T. Hesselberg und O. Devik. Braunschweig, Vieweg, 1913. 7+172 pp. M. 20.00

BLONDEL (A.). Sur la théorie des marées dans un canal. Application à la mer Rouge. (Thèse.) Toulouse, Privat, 1912. 4to. 58 pp.

BOGAERT (E. W.). L'effet gyrostatique et ses applications. Paris, Béranger, 1912. 241 pp. Fr. 10.00

BÖRNSTEIN (R.) und ROTH (W. A.). Physikalisch-chemische Tabellen. 4te umgearbeitete und vermehrte Auflage. Berlin, Springer, 1912. 8vo. 1330 pp. Moleskin. M. 56.00

BOUSSINESQ (J.). Complément à un récent mémoire intitulé "Sur les principes de la mécanique, etc." Paris, Gauthier-Villars, 1913. 4to. 49 pp. Fr. 3.00

BROGLIE (M. de). See THEORIE du rayonnement.

BROOKS (E. E.) and POYSER (A. W.). Magnetism and electricity; a manual for students in advanced classes. New York, Longmans, 1913. 8vo. 8+634 pp. $2.00

BROWN (H. T.). Cinq cent sept mouvements mécaniques. Traduit de l'anglais par H. Stévart. Liége, Desoer, 1913. 122 pp. Cloth. Fr. 2.00

CHOLLET (T.) et MINEUR (P.). Traité de géométrie descriptive. 1re partie. 8e édition. Paris, Vuibert, 1913. 8vo. 6+336 pp.

CLAUDEL (J.). L'introduction à la science de l'ingénieur. Partie théorique. Revue et mise au point par G. Dariés. 2 volumes. Paris, Dunod et Pinat, 1913. 8vo. 8+1858 pp. Cloth. Fr. 32.00

CROSS (W. E.). Elementary physical optics. New York, Oxford University Press, 1913. 12mo. 312 pp. $0.90

COULOM (F.). Cours élémentaire de géométrie descriptive. Paris, Delagrave, 1912. 18mo. 159 pp. Fr. 1.75

COURS de navigation et de compas. Paris, Challamel, 1913. 8vo. 8+371 pp.

DE HEEN (P.). Introduction à l'étude de la physique. La théorie des électrons et la théorie substantialiste. Bruxelles, Hayez, 1913. 287 pp. Fr. 12.00

EGERER (H.). Ingenieur-Mathematik. 1ter Band: Niedere Algebra und Analysis. Lineare Gebilde der Ebene und des Raumes in analytischer und vektorieller Behandlung. Kegelschnitte. Berlin, Springer, 1913. 8vo. 8+501 pp. Cloth. M. 12.00

FLEMING (J. A.). Propagation des courants électriques dans les conducteurs téléphoniques et télégraphiques. Traduit par C. Ravut. Paris, Gauthier-Villars, 1913. 8vo. 7+349 pp. Fr. 12.00

FRISCHAUF (J.). Die mathematischen Grundlagen der Landesaufnahme und Kartographie des Erdsphäroids. Stuttgart, Wittwer, 1913. 8vo. 12+192 pp. Cloth. M. 8.40

GANDILLOT (M.). Abrégé sur l'hélice et la résistance de l'air. Paris, Gauthier-Villars, 1912. 4to. 4+188 pp. Fr. 10.00

GIRARD (R.). Prontuario di tavole numeriche e formole per il computo metrico dei ponti in muratura, muri di sostegno e gallerie. Volume I. Roma, Bertero, 1913. 4to. 442 pp. L. 35.00

GOMMELET (J.). Manuel pratique du géomètre-expert. Paris, Dunod et Pinat, 1912. 18mo. Fr. 5.00

GRIGNON (A.). Traité de cosmographie. 8e édition. Paris, Vuibert, 1913. 8vo. 216 pp.

GUICHARD (C.). Traité de mécanique à l'usage des élèves de mathématiques A et B. 7e édition. Paris, Vuibert, 1913. 8vo. 7+248 pp.

HEATH (T. E.). Tracks of the sun and stars, A. D. 1900 to A. D. 37,900. Photographs from stereoscopic perspective drawings, showing in space of 3 dimensions the tracks of the sun and stars. London, Wesley, 1913. 8vo. 23 pp. Sewed. 5s.

HOUSTOUN (R. A.). An introduction to mathematical physics. New York, Longmans, 1913. 8vo. 10+199 pp. $2.00

KIRCHNER (F.). See BJERKNES (V.).

LAHARPE (de). Notes et formules de l'ingénieur. 17e édition. Paris, Geisler, 1913. 16mo. 2853+52 pp.

LANGEVIN (P.). See THÉORIE du rayonnement.

LAUE (M.). Das Relativitätsprinzip. 2te, vermehrte Auflage. (Die Wissenschaft, Band 38.) Braunschweig, Vieweg, 1913. 8vo. 12+272 pp. M. 8.80

LEDOUX (C.). Cours de géométrie, d'arpentage et de nivellement. Bruxelles, Lebègue, 1912. 7+223 pp. Cartonné. Fr. 2.50

LEGRAND (L.). La résistance de l'air envisagée comme base scientifique et expérimentale de l'aviation. Paris, Librairie aéronautique, 1912. 2+143 pp. Fr. 10.00

LE ROY (C.). Transport de force. Calculs techniques et économiques des lignes de transport et de distribution d'énergie électrique. 1re partie. Paris, Hermann, 1912. 8vo. 172 pp. Fr. 6.00

LÉVY (M.). La statique graphique et ses applications aux constructions. 2e édition. 2e partie. Paris, Gauthier-Villars, 1913. 8vo. 20+344 pp. Fr. 15.00

LITRE (E.). L'aviation et le mouvement terrestre. Théorie aérodynamique du virage, etc. Paris, Gauthier-Villars, 1912. 8vo. 4+48 pp. Fr. 2.00

LORENZ (H.). Nouvelle théorie et calcul des roues-turbines. Traduit de l'allemand par H. Espitallier et H. Strehler. Paris, Dunod et Pinat, 1913. 8vo. 14+312 pp. Cartonné. Fr. 14.00

MEZGER (C.). Die Chemie als mathematisches Problem. Mit 60 Strukturbildern im Text. Metz, Scriba, 1913. 8vo. 4+108 pp. M. 3.00

MINCHIN (G. M.). A treatise on hydrostatics; in 2 volumes. 2d edition, revised. London, Oxford University Press, 1913. 12mo. Volume 1, 204 pp., 4s. 6d. Volume 2, 204 pp., 6s.

MINEUR (P.). See CHOLLET (T.).

NACTERGAL (A.). Calcul des moments d'inertie. 2e édition revue et augmentée. Bruxelles, Bieleveld, 1912. 122 pp. Fr. 6.50

PENNEQUIN (L.). See BAKER (J. O.).

PIZZETTI (P.). Principî della teoria meccanica della figura dei pianeti. Pisa, Spoerri, 1913. 8vo. 13+251 pp. L. 12.00

POYSER (A. W.). See BROOKS (E. E.).

RAVUT (C.). See FLEMING (J. A.).

RECUEIL de constantes physiques. (Societé Française de Physique.) Publié par H. Abraham et P. Sacerdote, avec la collaboration d'un grand nombre de savants. Paris, Gauthier-Villars, 1913. 4to. 17+754 pp. Fr. 50.00

ROTH (W. A.). See BÖRNSTEIN (R.).

ROUSSELET (L.). Mécanique, électricité et construction appliquées aux appareils de levage. Tome II. Paris, Dunod et Pinat, 1913. 8vo. 6+752 pp. Cartonné. Fr. 45.00

ROY (L.). Sur la propagation des ondes dans les membranes flexibles. Paris, Gauthier-Villars, 1912. 4to. 101 pp. Fr. 5.00

SCHMIDT (F.). Beiträge zur Verteilung der Elektrizität auf zwei leitenden Kugeln, insbesondere für den Fall der Berührung. (Diss.) Halle, 1912. 4to. 46 pp.

SÉE (A.). En quoi consiste la stabilité. Paris, Gauthier-Villars, 1913. 18mo. 36 pp. Fr. 1.50

STÉVART (H.). See BROWN (H. T.).

TAFELN zur Ermittelung der in Zeitmass ausgedrückten Winkel aus den numerischen Werten der Kosinusfunktion. Berlin, Mittler, 1912. 8vo. 79 pp. M. 1.40

THÉORIE (La) du rayonnement et les quanta. Rapports et discussions de la réunion tenue à Bruxelles en 1911. Publiés par P. Langevin et M. de Broglie. Paris, Gauthier-Villars, 1912. 8vo. 468 pp. Fr. 15.00

VIOLEINE (A. P.). Nouvelles tables pour les calculs d'intérèts composés, d'annuités, et d'amortissements. 9e édition, entièrement refondue par A. Arnaudeau. Paris, Gauthier-Villars, 1913. 4to. Fr. 15.00

WIEGREFE (A.). Ueber einige mehrwertige Lösungen der Wellengleichung $\Delta u + k^2 u = 0$ und ihre Anwendung in der Beugungstheorie. (Diss.) Göttingen, 1912. 8vo. 95 pp.

TWENTY-SECOND ANNUAL LIST OF PAPERS

READ BEFORE THE AMERICAN MATHEMATICAL SOCIETY AND SUBSEQUENTLY PUBLISHED, INCLUDING REFERENCES TO THE PLACES OF THEIR PUBLICATION.

ALEXANDER, J. W., II. See VEBLEN, O.

BAKER, R. P. The Method of Monodromie with Applications to Three-Parameter Quartic Equations. Read (Chicago) April 5, 1912. *Annals of Mathematics*, vol. 14, No. 3, pp. 119–136; March, 1913.

BEAL, F. W. Normal Congruences Determined by Centers of Geodesic Curvature. Read Sept. 12, 1911. *American Journal of Mathematics*, vol. 35, No. 1, pp. 10–36; Jan., 1913.

BIRKHOFF, G. D. A Determinant Formula for the Number of Ways of Coloring a Map. Read April 27, 1912. *Annals of Mathematics*, vol. 14, No. 1, pp. 42–46; Sept., 1912.

—— Proof of Poincaré's Geometric Theorem. Read Oct. 26, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 14–22; Jan., 1913.

—— The Reducibility of Maps. Read April 27, 1912. *American Journal of Mathematics*, vol. 35, No. 2, pp. 115–128; April, 1913.

BLISS, G. A. Fundamental Existence Theorems. Read Dec. 28, 1907, (Chicago) Jan. 1, 1909, Sept. 15–17, 1909, Jan. 1, 1913. The Princeton Colloquium Lectures on Mathematics, pp. 1–107; New York, 1913.

BLUMBERG, H. Ueber algebraische Eigenschaften von linearen homogenen Differentialausdrücken. Read Sept. 10, 1912. Author's Dissertation, 53 pp.; Göttingen, 1912.

—— A Set of Postulates for Arithmetic and Algebra. Read Sept. 10, 1912. Proceedings of the Fifth International Congress of Mathematicians, vol. 2, pp. 461–465; 1913.

BÔCHER, M. A Simple Proof of a Fundamental Theorem in the Theory of Integral Equations. Read Dec. 28, 1910. *Annals of Mathematics*, vol. 14, No. 2, pp. 84–85; Dec., 1912.

BRADSHAW, J. W. An Infinite Product for $\pi/4$ Derived from Gregory's Series. Read Sept. 7, 1910. *Quarterly Journal of Pure and Applied Mathematics*, vol. 43, No. 4, pp. 378–379; July, 1912.

BRENKE, W. C. Transformation of Series by Means of Functions Admitting a Recurrent Relation. Read (Southwestern Section) Nov. 28, 1908, Nov. 27, 1909, and Nov. 26, 1910. *Annals of Mathematics*, vol. 13, No. 1, pp. 40–51; Sept., 1911.

CAJORI, F. Historical Note on the Graphic Representation of Imaginaries before the Time of Wessel. Read (Southwestern Section) Nov. 30, 1912. *American Mathematical Monthly*, vol. 19, Nos. 10–11, pp. 167–171; Oct.–Nov., 1912.

CAMP, B. H. Singular Multiple Integrals, with Applications to Series. Read April 27, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 42–64; Jan., 1913.

CARMICHAEL, R. D. Note on Multiply Perfect Numbers. Read April 28, 1911. *Proceedings of the Indiana Academy of Sciences*, pp. 257–270; 1912.

—— On the Theory of Relativity: Analysis of the Postulates. Read September 10, 1912. *Physical Review*, vol. 35, No. 3, pp. 153–176; Sept., 1912.

—— Generalizations of Euler's φ-Function, with Applications to Abelian Groups. Read (Southwestern Section) Dec. 2, 1911. *Quarterly Journal of Pure and Applied Mathematics*, vol. 44, Nos. 1–2, pp. 94–104; Oct., 1912,–Jan., 1913.

—— On the Remainder Term in a Certain Development of $f(a + x)$. Read Feb. 29, 1908. *American Mathematical Monthly*, vol. 20, No. 1, pp. 20–25; Jan., 1913.

—— On the Theory of Relativity: Mass, Force and Energy; Philosophical Aspects. Read (Southwestern Section) Nov. 30, 1912. *Physical Review*, ser. 2, vol. 1, Nos. 2, 3, pp. 161–197; Feb.–March, 1913.

—— Note on Fermat's Last Theorem. Read Dec. 31, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 5, pp. 233–236, and No. 8, pp. 402–403; Feb. and May, 1913.

—— Linear Mixed Equations and Their Analytic Solutions. Read Oct. 29, 1910. *American Journal of Mathematics*, vol. 35, No. 2, pp. 151–162; April, 1913.

—— On the Theory of Linear Difference Equations. Read (Chicago) April 6, 1912. *American Journal of Mathematics*, vol. 35, No. 2, pp. 163–182; April, 1913.

COBLE, A. B. An Application of Finite Geometry to the Characteristic Theory of the Odd and Even Theta Functions. Read Feb. 24, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 2, pp. 241–276; April, 1913.

COLE, F. N. The Triad Systems of Thirteen Letters. Read Sept. 11, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 1–5; Jan., 1913.

COOLIDGE, J. L. A Study of the Circle Cross. Read Dec. 31, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 2, pp. 149–174; April, 1913.

CRATHORNE, A. R. The Total Variation in the Isoperimetric Problem with Variable End Points. Read (Chicago) March 22, 1913. *Bulletin of the American Mathematical Society*, vol. 19, No. 10, pp. 519–522; July, 1913.

CURTISS, D. R. An Extension of Descartes' Rule of Signs. Read (Chicago) April 8, 1910, and April 5, 1912. *Mathematische Annalen*, vol. 73, No. 3, pp. 424–435; March, 1913.

DEDERICK, L. S. On the Character of a Transformation in the Neighborhood of a Point where its Jacobian Vanishes. Read Oct. 26, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 143–148; Jan., 1913.

DENTON, W. W. Projective Differential Geometry of Developable Surfaces. Read (Chicago) April 6, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 2, pp. 175–208; April, 1913.

DICKSON, L. E. Amicable Number Triples. Read Dec. 31, 1912. *American Mathematical Monthly*, vol. 20, No. 3, pp. 84–92; March, 1913.

DINES, L. L. The Highest Common Factor of a System of Polynomials in One Variable. Read April 28 and Oct. 28, 1911. *American Journal of Mathematics*, vol. 35, No. 2, pp. 129–150; April, 1913.

—— Concerning Two Recent Theorems on Implicit Functions. Read Oct. 26, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 9, pp. 462–467; June, 1913.

DODD, E. L. The Least Square Method Grounded with the Aid of an Orthogonal Transformation. Read Sept. 10, 1912. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 21, Nos. 8–9, pp. 177–183; Aug.–Sept., 1912.

—— The Probability Integral Deduced by Means of Developments in Finite Form. Read Jan. 1, 1913. *American Mathematical Monthly*, vol. 20, No. 4, pp. 123–127; April, 1913.

—— The Probability of the Arithmetic Mean Compared with That of Certain Other Functions of the Measurements. Read Sept. 10, 1912. *Annals of Mathematics*, vol. 14, No. 4, pp. 186–198; June, 1913.

—— An Erroneous Application of Bayes' Theorem to the Set of Real Numbers. Read Jan. 1, 1913. *Bulletin of the American Mathematical Society*, vol. 19, No. 9, pp. 479–482; June, 1913.

EIESLAND, J. On a Flat Spread-Sphere Geometry in Odd Dimensional Space. Read Dec. 27, 1911. *American Journal of Mathematics*, vol. 35, No. 2, pp. 201–228; April, 1913.

EISENHART, L. P. Minimal Surfaces in Euclidean Four-Space. Read Sept. 12, 1911. *American Journal of Mathematics*, vol. 34, No. 3, pp. 215–236; July, 1912.

—— Ruled Surfaces with Isotropic Generators. Read Dec. 28, 1911. *Rendiconti del Circolo Matematico di Palermo*, vol. 34, No. 1, pp. 29–40; July–Aug., 1912.

EMCH, A. On the Rectilinear Congruence Realizing a Circular Transformation of One Plane into Another. Read Sept. 12, 1911. *Annals of Mathematics*, vol. 13, No. 4, pp. 155–160; June, 1912.

—— Involutoric Circular Transformations as a Particular Case of the Steinerian Transformation and Their Invariant Nets of Cubics. Read (Chicago) Dec. 29, 1911. *Annals of Mathematics*, vol. 14, No. 2, pp. 57–71; Dec., 1912.

—— On Conformal Rational Transformations in a Plane. Read (Chicago) April 5, 1912. *Rendiconti del Circolo Matematico di Palermo*, vol. 34, No. 3, pp. 333–344; Nov.–Dec., 1912.

FIELD, P. On Coulomb's Laws of Friction. Read (Chicago) Dec. 29, 1911. *Zeitschrift für Mathematik und Physik*, vol. 61, Nos. 1–2, pp. 68–74; Dec., 1912.

FRIZELL, A. B. Foundations of Arithmetic. Read Dec. 28, 1905; (Southwestern Section) Nov. 28, 1908, (Chicago) Jan. 1, 2, 1909; (Southwestern Section) Nov. 27, 1909, Nov. 26, 1910. *Kansas University Science Bulletin*, vol. 5, No. 21, pp. 383–411; March, 1911.

GALAJIKIAN, H. On Certain Non-Linear Integral Equations. Read Dec. 31, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 7, pp. 342–346; April, 1913.

GLENN, O. E. Theorems on Reducible Quantics. Read Sept. 6, 1910, and Dec. 27, 1911. *Annals of Mathematics*, vol. 14, No. 1, pp. 27–41; Sept., 1912.

—— On the Structure of Forms and the Algebraical Theory of n Lines. Read Feb. 26, 1910. *American Journal of Mathematics*, vol. 34, No. 4, pp. 449–460; Oct., 1912.

GREEN, G. M. Projective Differential Geometry of Triple Systems of Surfaces. Read Feb. 22, 1913. Author's Dissertation, 28 pp.; Lancaster, May, 1913.

GRONWALL, T. H. On a Theorem of Fejér and an Analogon to Gibbs' Phenomenon. Read (Chicago) April 6, 1912. *Transactions of the American Mathematical Society*, vol. 13, No. 4, pp. 445–468; Oct., 1912.

—— Sur un Théorème de M. Picard. Read (Southwestern Section) Nov. 30, 1912. *Comptes Rendus de l'Académie des Sciences de Paris*, vol. 155, No. 17, pp. 764–766; Oct. 21, 1912.

—— On Analytic Functions of Constant Modulus on a Given Contour Read Sept. 11, 1912. *Annals of Mathematics*, vol. 14, No. 2, pp. 72–80; Dec., 1912.

—— Ueber das Verhalten der Riemannschen Zetafunktion auf der Geraden $\sigma = 1$. Read (Southwestern Section) Nov. 30, 1912. *Archiv der Mathematik und Physik*, ser. 3, vol. 20, No. 3, pp. 231–238; Jan., 1913.

—— Some Asymptotic Expressions in the Theory of Numbers. Read (Chicago) April 6, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 113–122; Jan., 1913.

—— Some Special Boundary Problems in the Theory of Harmonic Functions. Read Sept. 11, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 5, pp. 227–233; Feb., 1913.

—— Sur la Fonction $\zeta(s)$ de Riemann au Voisinage de $\sigma = 1$. Read (Southwestern Section) Nov. 30, 1912. *Rendiconti del Circolo Matematico di Palermo*, vol. 35, No. 1, pp. 95–102; Jan.–Feb., 1913.

—— Sur les Séries de Dirichlet Correspondant à des Caractères Complexes. Read (Southwestern Section) Nov. 30, 1912. *Rendiconti del Circolo Matematico di Palermo*, vol. 35, No. 2, pp. 145–159; March–April, 1913,

HILDEBRANDT, T. H. A Contribution to the Foundations of Fréchet's Calcul Fonctionnel. Read (Chicago) April 10, 1909. *American Journal of Mathematics*, vol. 34, No. 3, pp. 237–290; July, 1912.

—— Necessary and Sufficient Conditions for the Interchange of Limit and Summation in the Case of Sequences of Infinite Series of a Certain Type. Read (Chicago) April 6, 1912. *Annals of Mathematics*, vol. 14, No. 2, pp. 81–83; Dec., 1912.

HUNTINGTON, E. V. A Set of Postulates for Abstract Geometry, Expressed in Terms of the Simple Relation of Inclusion. Read Oct. 26, 1912. *Mathematische Annalen*, vol. 73, No. 4, pp. 522–559; May, 1913.

HURWITZ, W. A. On the Pseudo-Resolvent to the Kernel of an Integral Equation. Read Sept. 12, 1911. *Transactions of the American Mathematical Society*, vol. 13, No. 4, pp. 405–418; Oct., 1912.

INGOLD, L. Functional Differential Geometry. Read (Southwestern Section) Nov. 26, 1910. *Transactions of the American Mathematical Society*, vol. 13, No. 3, pp. 319–341; July, 1912.

JACKSON, D. On the Degree of Convergence of the Development of a Continuous Function According to Legendre's Polynomials. Read Feb. 24, 1912. *Transactions of the American Mathematical Society*, vol. 13, No. 3, pp. 305–318; July, 1912.

—— On Approximation by Trigonometric Sums and Polynomials. Read Feb. 24 and April 27, 1912. *Transactions of the American Mathematical Society*, vol. 13, No. 4, pp. 491–515; Oct., 1912.

JAMES, G. O. Reduction of Polaris Vertical Circle Observations for Time and Azimuth. Read (Southwestern Section) Nov. 27, 1909. *Popular Astronomy*, vol. 18, No. 2, pp. 92–100; Feb., 1910.

—— On the Relation of the Inertial and Empirical Trihedrons of Gravitational Astronomy. Read (Chicago) April 5, 1912. *Astronomical Journal*, vol. 27, Nos. 9–10, pp. 77–82; April 20, 1912.

KARPINSKI, L. C. Augrim Stones. Read Sept. 11, 1912. *Modern Language Notes*, vol. 27, No. 7, pp. 206–209; Nov., 1912.

—— Hindu Numerals among the Arabs. Read Jan. 1, 1913. *Bibliotheca Mathematica*, ser. 3, vol. 13, No. 2, pp. 97–98; May, 1913.

—— The Quadripartitum Numerorum of John of Meurs. Read (Chicago) April 5, 1912; and Jan. 1, 1913. *Bibliotheca Mathematica*, ser. 3, vol. 13, No. 2, pp. 99–114; May, 1913.

KASNER, E. Equitangential Congruences of Curves in Space. Read Feb. 25, 1911. *Rendiconti del Circolo Matematico di Palermo*, vol. 35, No. 3, pp. 283–285; May–June, 1913.

—— Differential-Geometric Aspects of Dynamics. Read Sept. 4, 1906, Sept. 15–17, 1909, April 30, 1910, and Oct. 29, 1910. The Princeton Colloquium Lectures on Mathematics, pp. 1–117; New York, 1913.

—— Conformal Geometry. Read Sept. 7, 1910. Proceedings of the Fifth International Congress of Mathematicians, vol. 2, pp. 81–87; 1913.

KEYSER, C. J. Concerning Multiple Interpretations of Postulate Systems and the Existence of Hyperspace. Read Jan. 1, 1913. *Journal of Philosophy, Psychology and Scientific Methods*, vol. 10, No. 10, pp. 253–267; May, 1913.

KILLAM, S. D. A Note on Graphical Integration of a Function of a Complex Variable. Read April 26, 1913. *Bulletin of the American Mathematical Society*, vol. 19, No. 10, pp. 522–524; July, 1913.

LAMOND, J. K. Improper Multiple Integrals over Iterable Fields. Read April 27, 1912. *Transactions of the American Mathematical Society*, vol. 13, No. 4, pp. 434–444; Oct., 1912.

LEFSCHETZ, S. Double Curves of Surfaces Projected from Space of Four Dimensions. Read Sept. 10, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 2, pp. 70–74; Nov., 1912.

—— Two Theorems on Conics. Read (Southwestern Section) Dec. 2, 1911. *Annals of Mathematics*, vol. 14, No. 2, pp. 47–50; Dec., 1912.

—— On the Existence of Loci with Given Singularities. Read Sept. 12, 1911. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 23–41; Jan., 1913.

—— On Some Topological Properties of Plane Curves and a Theorem of Möbius. Read Sept. 12, 1911. *American Journal of Mathematics*, vol. 35, No. 2, pp. 189–200; April, 1913.

LEHMER, D. N. Certain Theorems in the Theory of Quadratic Residues. Read (San Francisco Section) Sept. 28, 1907. *American Mathematical Monthly*, vol. 20, No. 5, pp. 151–157; May, 1913.

LENNES, N. J. Concerning Van Vleck's Non-Measurable Set. Read April 27, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 109–112; Jan., 1913.

MACGREGOR, H. H. Three-Dimensional Chains and the Associated Collineations in Space. Read (Chicago) Dec. 31, 1909. *Annals of Mathematics*, vol. 14, No. 1, pp. 1–13; Sept., 1912.

MACMILLAN, W. D. A Reduction of a System of Power Series to an Equivalent System of Polynomials. Read (Chicago) Dec. 29, 1910, and April 28, 1911. *Mathematische Annalen*, vol. 72, No. 2, pp. 157–179; May, 1912.

—— A Method for Determining the Solutions of a System of Analytic Functions in the Neighborhood of a Branch Point. Read April 28, 1911. *Mathematische Annalen*, vol. 72, No. 2, pp. 180–202; May, 1912.

—— On Poincaré's Correction to Bruns' Theorem. Read Jan. 2, 1913. *Bulletin of the American Mathematical Society*, vol. 19, No. 7, pp. 349–355; April, 1913.

MACNEISH, H. F. Linear Polars of the k-Hedron in n-Space. Read Dec. 30, 1909. Author's Dissertation, v+25 pp.; Chicago, March, 1912.

MARSHALL, E. R. A Labor Saving Device for Serial Multiplication or Division by means of an Arithmometer in Cases of Small Differences of Consecutive Results. Read April 27, 1912. *American Mathematical Monthly*, vol. 19, Nos. 8–9, pp. 141–152; Aug.-Sept., 1912.

MILES, E. J. Determination of the Constants in Euler's Problem Concerning the Minimum Area Between a Curve and Its Evolute. Read (Chicago) Jan. 1, 1909. *Annals of Mathematics*, vol. 14, No. 1, pp. 14–26; Sept., 1912.

—— Surfaces of Revolution of Minimum Resistance. Read Jan. 1, 1909. *Bulletin of the American Mathematical Society*, vol. 19, No. 1, pp. 1–12; Oct., 1912.

MILLER, G. A. Some Properties of the Group of Isomorphisms. Read Sept. 12, 1911. *Archiv der Mathematik und Physik*, ser. 3, vol. 19, No. 4, pp. 295–301; June, 1912.

—— Infinite Systems of Indivisible Groups. Read (Chicago) April 5, 1912. *Transactions of the American Mathematical Society*, vol. 13, No. 4, pp. 419–433; Oct., 1912.

—— A Few Theorems Relating to Sylow Subgroups. Read Sept. 10, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 2, pp. 63–66; Nov., 1912.

—— Gauss's Lemma and Some Related Group Theory. Read (Chicago) Dec. 30, 1911. *Prace Matematyczno-Fizyczne*, vol. 23, pp. 25–29; 1912.

—— Groups Containing a Given Number of Operators Whose Orders are Powers of the Same Prime Number. Read Dec. 28, 1911. *American Journal of Mathematics*, vol. 35, No. 1, pp. 1–9; Jan., 1913.

—— Second Note on the Groups Generated by Operators Transforming Each Other into Their Inverses. Read Feb. 24, 1912. *Quarterly Journal of Pure and Applied Mathematics*, vol. 44, No. 2, pp. 142–146; Jan., 1913.

—— The Product of Two or More Groups. Read Dec. 31, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 6, pp. 303–310; March, 1913.

MITCHELL, H. H. Determination of the Finite Quaternary Linear Groups. Read Oct. 28, 1911, and Feb. 24, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 123–142; Jan., 1913.

MOORE, C. L. E. See PHILLIPS, H. B.

MOORE, C. N. On Convergence Factors in Double Series and the Double Fourier Series. Read April 29 and Dec. 27, 1911. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 73–104; Jan., 1913.

MOORE, R. L. On Duhamel's Theorem. Read (Chicago) April 9, 1910. *Annals of Mathematics*, vol. 13, No. 4, pp. 161–166; June, 1912.

MOULTON, F. R. Closed Orbits of Ejection and Related Periodic Orbits. Painlevé's Theorem. Read (Chicago) April 6, 1912. *Proceedings of the London Mathematical Society*, ser. 2, vol. 11, Nos. 4–5, pp. 367–397; Oct., 1912.

—— Periodic Oscillating Satellites in the Problem of Three Bodies. Read (Chicago) April 10, 1909. *Mathematische Annalen*, vol. 73, No. 4, pp. 441–479; May, 1913.

—— Relations among Families of Periodic Orbits in the Restricted Problem of Three Bodies. Read (Chicago) Dec. 29, 1911. Proceedings of the Fifth International Congress of Mathematicians, vol. 2, pp. 182–187; 1913.

NEWSON, H. B. Theory of Collineations. Read (Southwestern Section) Dec. 1, 1906, Nov. 30, 1907; Sept. 11, 1908, Sept. 14, 1909; (Southwestern Section) Nov. 27, 1909. *Kansas University Science Bulletin*, vol. 6, No. 1, pp. 1–319; Dec., 1911.

OSGOOD, W. F. Existenzbeweis betreffend Funktionen, welche zu einer eigentlich diskontinuierlichen automorphen Gruppe gehören. Read Oct. 26, 1912. *Rendiconti del Circolo Matematico di Palermo*, vol. 35, No. 1, pp. 103–106; Jan.–Feb., 1913.

OSGOOD, W. F., and TAYLOR, E. H. Conformal Transformations on the Boundaries of Their Regions of Definition. Read Dec. 30, 1902, and April 30, 1910. *Transactions of the American Mathematical Society*, vol. 14, No. 2, pp. 277–298; April, 1913.

PHILLIPS, H. B., and MOORE, C. L. E. A Theory of Linear Distance and Angle. Read April 27, 1912. *Proceedings of the American Academy of Arts and Sciences*, vol. 48, No. 3, pp. 45–80; July, 1912.

PITCHER, A. D. Concerning the Property Δ of a Class of Functions. Read April 26, 1913. *Bulletin of the American Mathematical Society*, vol. 19, No. 9, pp. 468–472; June, 1913.

RANUM, A. On the Projective Differential Geometry of n-Dimensional Spreads Generated by ∞^1 Flats. Read Sept. 12 and Dec. 27, 1911. *Annali di Matematica Pura ed Applicata*, ser. 3, vol. 19, No. 3, pp. 205–249; Sept., 1912.

—— Lobachefskian Polygons Trigonometrically Equivalent to the Triangle. Read Sept. 10, 1912. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 21, Nos. 10–12, pp. 228–240; Oct.–Dec., 1912.

REDDICK, H. W. Systems of Plane Curves Whose Intrinsic Equations are Analogous to the Intrinsic Equation of an Isothermal System. Read Oct. 26, 1912. *Annals of Mathematics*, vol. 14, No. 4, pp. 179–185; June, 1913.

RICHARDSON, R. G. D. Ueber die notwendigen und hinreichenden Bedingungen für das Bestehen eines Kleinschen Oszillationstheorems. Read Dec. 28, 1911. *Mathematische Annalen*, vol. 73, No. 2, pp. 289–304; Jan., 1913.

ROEVER, W. H. The Southerly and Easterly Deviations of Falling Bodies for an Unsymmetrical Gravitational Field of Force. Read (Southwestern Section) Dec. 2, 1911, and (Chicago) April 5, 1912. *Transactions of the American Mathematical Society*, vol. 13, No. 4, pp. 469–490; Oct., 1912.

ROWE, J. E. Bicombinants of the Rational Plane Quartic and Combinants of the Rational Plane Quintic. Read Sept. 12, 1911. *Transactions of the American Mathematical Society*, vol. 13, No. 3, pp. 387–404; July, 1912.

—— Three or More Rational Curves Collinearly Related. Read Dec. 31, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 8, pp. 395–401; May, 1913.

—— Cusp and Undulation Invariants of Rational Curves. Read Feb. 24, 1912. *Annals of Mathematics*, vol. 14, No. 4, pp. 199–210; June, 1913.

RUNGE, C. Graphische Lösung von Randwertaufgaben der Gleichung $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = 0$. Read Oct. 30, 1909. *Göttinger Nachrichten*, 1911, pp. 443–448.

SCHWEITZER, A. R. Theorems on Functional Equations. Read April 27, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 2, pp. 66–70; Nov., 1912.

—— A Theory of Geometrical Relations. Read Sept. 3, 1906, and (Chicago) March 30, 1907. *American Journal of Mathematics*, vol. 35, No. 1, pp. 37–56; Jan., 1913.

SHAW, J. B. Quaternion Developments with Applications. Read (Chicago) April 8, 1910, and April 28, 1911. *Transactions of the American Mathematical Society*, vol. 13, No. 3, pp. 279–292; July, 1912.

SICELOFF, L. P. Simple Groups from Order 2001 to Order 3640. Read Oct. 28, 1911. *American Journal of Mathematics*, vol. 34, No. 4, pp. 361–372; Oct., 1912.

SILVERMAN, L. L. On the Definition of the Sum of a Divergent Series. Read Oct. 29, 1910, Dec. 28, 1911, and Jan. 2, 1913. *University of Missouri Studies, Mathematics Series*, vol. 1, No. 1, pp. 1–96; April, 1913.

SLOCUM, S. E. A General Formula for the Torsional Deflection of Shafts. Read Dec. 28, 1911. *Journal of the Franklin Institute*, vol. 174, No. 1, pp. 83–89; July, 1912.

SMAIL, L. L. Some Generalizations in the Theory of Summable Divergent Series. Read Jan. 2, 1913. Author's Dissertation, 46 pp.; Lancaster, May, 1913.

SNYDER, V. Algebraic Surfaces Invariant under an Infinite Discontinuous Group of Birational Transformations. Second Paper. Read Jan. 2, 1913. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 105–108; Jan., 1913.

STOUFFER, E. B. Invariants of Linear Differential Equations with Applications to Ruled Surfaces in Five-Dimensional Space. Read April 29, 1911. *Proceedings of the London Mathematical Society*, ser. 2, vol. 11, No. 3, pp. 185–224; July, 1912.

TAYLOR, E. H. An Extension of a Theorem of Painlevé. Read Oct. 26, 1912. *Bulletin of the American Mathematical Society*, vol. 19, No. 8, pp. 403–406; May, 1913.

—— See OSGOOD, W. F.

VANDIVER, H. S. Theory of Finite Algebras. Read (Chicago) Dec. 29, 1911. *Transactions of the American Mathematical Society*, vol. 13, No. 3, pp. 293–304; July, 1912.

VAN VLECK, E. B. On the Extension of a Theorem of Poincaré for Difference Equations. Read April 29, 1911. *Transactions of the American Mathematical Society*, vol. 13, No. 3, pp. 342–352; July, 1912.

—— One-Parameter Projective Groups and the Classification of Collineations. Read April 29, 1911. *Transactions of the American Mathematical Society*, vol. 13, No. 3, pp. 353–386; July, 1912.

VEBLEN, O. An Application of Modular Equations in Analysis Situs. Read April 27, 1912. *Annals of Mathematics*, vol. 14, No. 2, pp. 86–94; Dec., 1912.

—— Decomposition of an n-Space by a Polyhedron. Read Sept. 11, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 65–72; Jan., 1913.

VEBLEN, O., and ALEXANDER, J. W., II. Manifolds of n Dimensions. Read Feb. 22, 1913. *Annals of Mathematics*, vol. 14, No. 4, pp. 163–178; June, 1913.

WHITE, H. S. Triple Systems as Transformations, and Their Paths among Triads. Read Sept. 11, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 1, pp. 6–13; Jan., 1913.

WILCZYNSKI, E. J. A Forgotten Theorem of Newton on Planetary Motion and an Instrumental Solution of Kepler's Problem. Read (Chicago) April 5, 1912. *Astronomical Journal*, vol. 27, No. 20, pp. 155–156; Jan. 31, 1913.

WILLIAMS, F. B. Curves on Quintic Scrolls. Read (Chicago) Dec. 30, 1910. *American Journal of Mathematics*, vol. 34, No. 4, pp. 421–448; Oct., 1912.

WILLIAMS, K. P. The Solutions of Non-Homogeneous Linear Difference Equations and Their Asymptotic Form. Read Feb. 24, 1912. *Transactions of the American Mathematical Society*, vol. 14, No. 2, pp. 209–240; April, 1913.

—— The Asymptotic Form of the Function $\Psi(x)$. Read April 26, 1913. *Bulletin of the American Mathematical Society*, vol. 19, No. 9, pp. 472–479; June, 1913.

WILSON, E. B. Some Mathematical Aspects of Relativity. Read Dec. 27, 1911. *Proceedings of the American Academy of Arts and Sciences*, vol. 48, No. 11, pp. 389–507; Nov., 1912.

YOUNG, A. E. On Certain Orthogonal Systems of Lines and the Problem of Determining Surfaces Referred to Them. Read April 28, 1911. *American Journal of Mathematics*, vol. 34, No. 4, pp. 373–382; Oct., 1912.

INDEX OF VOLUME XIX.

ARCHIBALD, R. C. See REVIEWS, under Halsted, Saint-Laguë.

BORGER, R. L. See REVIEWS, under Kowalewski.

BROWN, E. W. See REVIEWS, under Annuaire.

BYERLY, W. E. See REVIEWS, under Wilson.

CARMICHAEL, R. D. Note on Fermat's Last Theorem, 233.

—— Second Note on Fermat's Last Theorem, 402.

—— See REVIEWS, under Forsyth, Poincaré, Voigt.

COLE, F. N. Reports of Meetings of the American Mathematical Society: Nineteenth Summer Meeting, 51; October Meeting, 163; Nineteenth Annual Meeting, 275; February Meeting, 387; April Meeting, 497.

COOLIDGE, J. L. See REVIEWS, under Schulze.

COWLEY, E. B. See REVIEWS, under Kommerell.

CRATHORNE, A. R. The Total Variation in the Isoperimetric Problem with Variable End Points, 519.

—— See REVIEWS, under Horn, Scheffers.

CUMMINGS, L. D. Note on the Groups for Triple Systems, 355.

DINES, L. L. Concerning Two Recent Theorems on Implicit Functions, 462.

DODD, E. L. An Erroneous Application of Bayes' Theorem to the Set of Real Numbers, 479.

DOWLING, L. W. See REVIEWS, under Boehm.

DRESDEN, A. See REVIEWS, under Love.

EMCH, A. See REVIEWS, under Study.

FRIZELL, A. B. The Fifth International Congress of Mathematicians. Section I: Arithmetic, Algebra, Analysis, 131.

GALAJIKIAN, H. On Certain Non-Linear Integral Equations, 342.

GILLESPIE, D. C. See REVIEWS, under Hedrick.

GRONWALL, T. H. Some Special Boundary Problems in the Theory of Harmonic Functions, 227.

GROVE, C. C. See REVIEWS, under Jacob, Richard.

HURWITZ, W. A. See REVIEWS, under Kneser.

KASNER, E. See REVIEWS, under Riquier.

KILLAM, S. D. A Note on Graphical Integration of a Function of a Complex Variable, 522.

LAVES, K. See REVIEWS, under Haret, Schwahn.

LEFSCHETZ, S. Double Curves of Surfaces Projected from Space of Four Dimensions, 70.

LEIB, D. D. See REVIEWS, under Hulburt.

LENNES, N. J. See REVIEWS, under Townsend.

LONGLEY, W. R. See REVIEWS, under Heywood, Lalesco.

LUNN, A. C. See REVIEWS, under Carvallo, King, Markoff.

MCKELVEY, J. V. See REVIEWS, under Hawkes, Hime.

MACMILLAN, W. D. On Poincaré's Correction to Bruns' Theorem, 349.

MANNING, W. A. The Twenty-Third Regular Meeting of the San Francisco Section, 517.
MILES, E. J. Surfaces of Revolution of Miminum Resistance, 1.
MILLER, G. A. A Few Theorems Relating to Sylow Subgroups, 63.
—— The Product of Two or More Groups, 303.
—— See REVIEWS, under Darboux, Séguier, Vivanti.
MYERS, G. W. See REVIEWS, under Dziobek, Hack.
PIERPONT, J. See REVIEWS, under Hermite.
PITCHER, A. D. Concerning the Property Δ of a Class of Functions, 468.
PONZER, E. W. See REVIEWS, under Auerbach, Internationale Mathematische Unterrichtskommission, Vahlen.
POOR, V. C. A Theorem on Asymptotic Series, 346.
PUTNAM, T. M. The Twenty-Second Regular Meeting of the San Francisco Section, 174.
RANUM, A. See REVIEWS, under Bonola.
ROWE, J. E. Three or More Rational Curves Collinearly Related, 395.
SCHWEITZER, A. R. Theorems on Functional Equations, 66.
SHAW, J. B. See REVIEWS, under Coffin, Föppl, Korn, Poincaré, Weber.
SISAM, C. H. See REVIEWS, under Epstein.
SLAUGHT, H. E. The Spring Meeting of the Chicago Section, 441.
SMITH, D. E. See REVIEWS, under Fagnano, Heath, Heiberg, Höfler, Mannoury, Rangācārya, Tannery.
SNYDER, V. The Fifth International Congress of Mathematicians, Cambridge, 1912, 107.
—— The Fifth International Congress of Mathematicians. Sections II–IV, 175.
—— The Münster Meeting of the Deutsche Mathematiker-Vereinigung, 191.
—— See REVIEWS, under Salmon.
TAYLOR, E. H. An Extension of a Theorem of Painlevé, 403.
VAN DER VRIES, J. N. The Sixth Regular Meeting of the Southwestern Section, 219.
WESTLUND, J. See REVIEWS, under Smith.
WHITE, H. S. See REVIEWS, under Beutel.
WILCZYNSKI, E. J. Some General Aspects of Modern Geometry, 331.
WILLIAMS, K. P. The Asymptotic Form of the Function $\Psi(x)$, 472.
WILSON, E. B. The Unification of Vectorial Notations, 524.
—— See REVIEWS, under Brill, Cosserat, Föppl, Krause, Lamb, Mills, Orlich, Prasad, Southall, Whittaker.
YOUNG, J. W. See REVIEWS, under Enriques, Natorp, Volkmann.

REVIEWS.

Abraham, M. See Föppl, A.
Annuaire du Bureau des Longitudes pour l'An 1913, E. W. BROWN, 428.
Auerbach, F., and Rothe, R. Taschenbuch für Mathematiker und Physiker, 1911, E. W. PONZER, 21.
Barbarin, P. See Halsted, G. B.
Beutel, E. Algebraische Kurven, Zweiter Teil: Theorie und Kurven dritter und vierter Ordnung, H. S. WHITE, 417.

Boehm, K. Elliptische Funktionen, Zweiter Teil, L. W. Dowling, 146.

Bonola, R. Non-Euclidean Geometry (English translation by H. S. Carslaw), A. Ranum, 22.

Brill, A. Das Relativitätsprinzip. Eine Einführung in die Theorie, E. B. Wilson, 321.

Cahen, A. See Vivanti, G.

Carslaw, H. S. See Bonola, R.

Carvallo, E. Calcul des Probabilités et ses Applications, A. C. Lunn, 535.

Coffin, J. G. Vector Analysis (second edition), J. B. Shaw, 18.

Cosserat, E. and F. Théorie des Corps déformables, E. B. Wilson, 242.

Darboux, G. Eloges académiques et Discours, G. A. Miller, 147.

Dziobek, O. Differential- und Integralrechnung, G. W. Myers, 318.

Enriques, F. Probleme der Wissenschaft. Part I: Wirklichkeit und Logik; Part II: Die Grundbegriffe der Wissenschaft (Deutsche Uebersetzung von K. Grelling), J. W. Young, 370.

Epstein, P., and Timerding, H. E. Repertorium der höheren Mathematik, erster Band, erste Hälfte; zweiter Band, erste Hälfte, (zweite Auflage), C. H. Sisam, 372.

Fagnano, G. C. dei T. di. Opere Matematiche (edited by V. Volterra, G. Loria, and D. Gambioli), D. E. Smith, 246.

Föppl, A. Technische Mechanik. Erster Band: Einführung in die Mechanik (vierte Auflage), E. B. Wilson, 321.

—— Theorie der Elektrizität. Erster Band: Einführung in die Maxwellsche Theorie der Elektrizität (vierte Auflage, herausgegeben von M. Abraham), J. B. Shaw, 483.

Forsyth, A. R. Lehrbuch der Differentialgleichrungen (Deutsche Uebersetzung von W. Jacobsthal, zweite Auflage), R. D. Carmichael, 256.

Fréchet, M. See Heywood, H. B.

Gambioli, D. See Fagnano, G. C. dei T. di.

Goodenough, G. A. See Townsend, E. J.

Granville, W. A. See Smith, P. F.

Grelling, K. See Enriques, F.

Hack, F. Wahrscheinlichkeitsrechnuhg, G. W. Myers, 319.

Halsted, G. B. Géométrie rationelle, Traité élémentaire de la Science de l'Espace (traduction française par P. Barbarin), R. C. Archibald, 23.

Haret, S. C. Mécanique sociale, K. Laves, 32.

Hawkes, H. E., Luby, W. A., and Touton, F. C. Second Course in Algebra, J. V. McKelvey, 29.

Heath, T. L. The Method of Archimedes Recently Discovered by Heiberg, D. E. Smith, 248.

Hedrick, E. R., and Kellogg, O. D. Applications of the Calculus to Mechanics, D. C. Gillespie, 148.

Heiberg, J. L. Naturwissenschaften und Mathematik im klassischen Altertum, D. E. Smith, 84.

—— See Tannery, P.

Hermite, C. Oeuvres, Volume III (published by E. Picard), J. Pierpont, 83.

Heywood, H. B., and Fréchet, M. L'Equation de Fredholm et ses Applications à la Physique mathématique, W. R. Longley, 236.

Hime, H. W. L. Anharmonic Coordinates, J. V. McKelvey, 416.

Höfler, A. Didaktik des mathematischen Unterrichts, D. E. Smith, 249.

Horn, J. Einführung in die Theorie der partiellen Differentialgleichungen, A. R. CRATHORNE, 254.

Hulburt, L. S. Differential and Integral Calculus, D. D. LEIB, 197.

Internationale Mathematische Unterrichtskommission. Berichte und Mitteilungen, Hefte IV–VII, E. W. PONZER, 20.

Jacob, L. Le Calcul mécanique, C. C. GROVE, 30.

Jacobsthal, W. See Forsyth, A. R.

Kellogg, O. D. See Hedrick, E. R.

King, W. I. Elements of Statistical Method, A. C. LUNN, 535.

Kneser, A. Die Integralgleichungen und ihre Anwendungen in der mathematischen Physik, W. A. HURWITZ, 406.

Kommerell, V. and K. Spezielle Flächen und Theorie der Strahlensysteme, E. B. COWLEY, 253.

Korn, A. Ueber freie und erzwungene Schwingungen, J. B. SHAW, 88.

Kowalewski, G. Grundzüge der Differential- und Integralrechnung, R. L. BORGER, 531.

Krause, M. Theorie der elliptischen Funktionen (unter Mitwirkung von E. Naetsch), E. B. WILSON, 427.

Lalesco, T. Introduction à la Théorie des Equations intégrales, W. R. LONGLEY, 236.

Lamb, H. Dynamical Theory of Sound, E. B. WILSON, 260.

Liebmann, H. See Markoff, A. A.

Loria, G. See Fagnano, G. C. dei T. di.

Love, A. E. H. Differential and Integral Calculus, A. DRESDEN, 374.

Luby, W. A. See Hawkes, H. E.

Mannoury, G. Methodologisches und Philosophisches zur Elementar-Mathematik, D. E. SMITH, 86.

Markoff, A. A. Wahrscheinlichkeitsrechnung (zweite Auflage, übersetzt von H. Liebmann), A. C. LUNN, 535.

Mills, J. Introduction to Thermodynamics, E. B. WILSON, 428.

Natorp, P. Die logischen Grundlagen der exakten Wissenschaften, J. W. YOUNG, 370.

Naetsch, E. See Krause, M.

Orlich, E. Theorie der Wechselströme, E. B. WILSON, 321.

Picard, E. See Hermite, C.

Poincaré, H. Der Wert der Wissenschaft (Uebertragen von E. Weber, zweite Auflage), J. B. SHAW, 252.

—— Calcul des Probabilités (rédaction von A. Quiquet, deuxième édition), R. D. CARMICHAEL, 259.

Prasad, G. Differential Calculus. Integral Calculus, E. B. WILSON, 363.

Quiquet, A. See Poincaré, H.

Rangācārya, M. The Ganita-Sāra-Sangraha of Mahāvīrācārya, D. E. SMITH, 310.

Richard, P. J. Etude sur l'Assurance complémentaire de l'Assurance sur la Vie, C. C. GROVE, 89.

Riquier, C. Les Systèmes d'Equations aux Dérivées partielles, E. KASNER, 12.

Rogers, R. A. P. See Salmon, G.

Rothe, R. See Auerbach, F.

Sainte-Laguë, A. Notions de Mathématiques, R. C. ARCHIBALD, 421.

Salmon, G. Analytic Geometry of Three Dimensions (fifth edition, revised by R. A. P. Rogers), Vol. I, V. SNYDER, 80.

Scheffers, G. Lehrbuch der Mathematik fur Studierende der Naturwissenschaften und der Technik (zweite Auflage), A. R. CRATHORNE, 419.

Schulze, A. The Teaching of Mathematics in Secondary Schools, J. L. COOLIDGE, 411.

Schwahn, P. Mathematische Theorie der astronomischen Finsternisse, K. LAVES, 31.

Séguier, J. A. de. Théorie des Groupes finis. Eléments de la Théorie des Groupes de Substitutions, G. A. MILLER, 356.

Smith, P. F., and Granville, W. A. Elementary Analysis, J. WESTLUND, 27.

Southall, J. P. C. The Principles and Methods of Geometrical Optics, E. B. WILSON, 74.

Study, E. Vorlesungen über ausgewählte Gegenstände der Geometrie, erstes Heft: Ebene analytische Kurven und zu ihnen gehörige Abbildungen, A. EMCH, 15.

Tannery, P. Mémoires scientifiques (publiés par J. L. Heiberg et H. G. Zeuthen), Vol. I: Sciences exactes dans l'Antiquité, D. E. SMITH, 367.

Timerding, H. E. See Epstein, P.

Touton, F. C. See Hawkes, H. E.

Townsend, E. J., and Goodenough, G. A. First Course in Calculus. Essentials of Calculus, N. J. LENNES, 316.

Vahlen, T. Konstruktionen und Approximationen, E. W. PONZER, 92.

Vivanti, G. Les Fonctions polyédriques et modulaires (traduit par A. Cahen), G. A. MILLER, 534.

Voigt, A. Theorie der Zahlenreihen und der Reihengleichungen, R. D. CARMICHAEL, 203.

Volkmann, P. Erkenntnistheoretische Grundzüge der Naturwissenschaften (zweite Auflage), J. W. YOUNG, 370.

Volterra, V. See Fagnano, G. C. dei T. di.

Weber, E. See Poincaré, H.

Weber, H. Die partiellen Differentialgleichungen der mathematischen Physik (fünfte Auflage, zweiter Band), J. B. SHAW, 482.

Weber, H., und Wellstein, J. Encyklopädie der Elementar-Mathematik, Band III, J. B. SHAW, 87, 422.

Wellstein, J. See Weber, H.

Whittaker, E. T. A History of the Theories of the Aether and Electricity from the Age of Descartes to the Close of the Nineteenth Century, E. B. WILSON, 423.

Wilson, E. B. Advanced Calculus, W. E. BYERLY, 360.

Zeuthen, H. G. See Tannery, P.

Index of Volume XIX, 559.
New Publications, 44, 102, 156, 212, 268, 325, 380, 435, 489, 541.
Notes, 33, 93, 149, 204, 264, 322, 375, 429, 484, 538.
Papers Read before the Society and Subsequently Published, Twenty-Second Annual List of, 550.

NOTES AND OTHER ITEMS.

Academies, Associations, Congresses, and Societies:
American Mathematical Society: Annual Meeting, 93, 498; Chicago Section, 94; Election of Officers, 164, 276; New Members Admitted, 51, 163, 276, 387, 497; Princeton Colloquium Lectures, 387, 484; Reports and Statistics, 276; Southwestern Section, 93; Summer Meeting and Colloquium, 94, 429; Transactions, 33, 149, 264, 484, 497.
Associations for the Advancement of Science: American, 94, 149, 265; Australasian, 95; British, 95, 430; French, 149, 539; Italian, 150.
Associations of Teachers of Mathematics: International Commission on the Teaching of Mathematics, 95.
American Academy of Arts and Sciences, 265; Belgian Academy, 538; Berlin Academy, 39; Bologna Academy, 486; Committee on Potential and Elasticity Notation, 538; Edinburgh Mathematical Society, 94, 205; Italian Society of Sciences, 40, 154; Kasan Mathematical Society, 379, 485; London Mathematical Society, 205, 265, 322, 376, 430; Paris Academy, 39, 153, 323, 430, 485, 538; Royal Society, 211; Society for the Promotion of Engineering Education, 33; Swiss Mathematical Society, 205, 430.
Books, Announcements of New, 33, 34, 34, 39, 206, 265, 322.
Catalogues of Books, Models, etc., 43, 96, 155, 206, 206, 267, 380, 485, 485.
Doctorates in Mathematics, American, 34; French, 431; German, 206.
Journals:
American Journal of Mathematics, 33, 94, 322, 484; American Mathematical Monthly, 204; Annals of Mathematics, 33, 94, 264, 375, 538; Die Naturwissenschaften, 376; L'Enseignement Mathématique, 323; Proceedings of the American Academy of Arts and Sciences, 265; Transactions of the American Mathematical Society, 33, 149, 264, 484.

Papers and Communications Presented to the Society, Authors:
Alexander, J. W., II., 388.
Altshiller, N., 498.
Baker, R. P., 442, 442.
Batchelder, P. M., 498, 498.
Bateman, H., 388, 388.
Bauer, G. N., 443.
Beal, W. O., 442.
Bell, E. T., 164, 164.
Bennett, A. A., 164.
Bennett, E. R., 219, 219.
Bernstein, B. A., 517.
Bill, E. G., 498.
Birkhoff, G. D., 164, 498, 499.
Blichfeldt, H. F., 517.
Bliss, G. A., 278, 443.
Blumberg, H., 52, 52.
Bôcher, M., 444, 498.
Buchanan, D., 278.
Cajori, F., 219.
Camp, B. H., 388.
Caparo, J. A., 278.
Carmichael, R. D., 51, 219, 277, 277, 277, 443, 443, 443.
Chittenden, E. W., 443, 443.
Clements, G. R., 164.
Cole, F. N., 52.
Conner, J. R., 278, 278.
Coolidge, J. L., 278.
Copeland, L. P., 388.
Craig, C. F., 498.
Crathorne, A. R., 444.
Curtiss, D. R., 279.
Dederick, L. S., 164.
Dickson, L. E., 278, 278, 443, 443, 443.
Dines, L. L., 164, 164.
Dodd, E. L., 52, 52, 220, 278, 279, 443, 498.
Eiesland, J., 278.
Eisenhart, L. P., 279, 499.
Emch, A., 219, 278.
Ettlinger, H. J., 498.
Evans, G. C., 278.
Field, P., 278.
Fischer, C. A., 388, 499.
Fite, W. B., 280.
Fréchet, M., 498.
Frizell, A. B., 279.
Gaba, M. G., 499.
Galajikian, H., 278.
Glenn, O. E., 53, 444.
Green, G. M., 388, 499.
Gronwall, T. H., 52, 53, 53, 220, 220, 220, 220, 279, 279, 388, 388, 388, 442, 498.
Hancock, H., 279, 279, 388.

Hedrick, E. R., 220.
Hennel, C. B., 279.
Huntington, E. V., 164, 279, 499.
Hurwitz, W. A., 277, 278, 499.
Ingold, L., 220.
Jackson, D., 52, 164, 388.
Karpinski, L. C., 52, 279, 279, 442, 442.
Kasner, E., 164, 279, 388, 499.
Kellogg, O. D., 444.
Keyser, C. J., 279.
Killam, S. D., 499.
Laves, K., 444.
Lefschetz, S., 52, 220, 443.
Lehmer, D. N., 174.
Lennes, N. J., 388, 388.
Lewis, C. I., 174.
Lipke, J., 279.
Love, C. E., 280, 443.
Lunn, A. C., 443, 443.
McDonald, J., 443.
MacMillan, W. D., 280, 280.
Miller, G. A., 52, 278, 442.
Miser, W. L., 442.
Mitchell, H. H., 52, 499.
Moore, E. H., 278, 444.
Moore, R. L., 498.
Morgan, F. M., 52, 278, 499.
Moulton, F. R., 443.
Neikirk, L. I., 517.
Osgood, W. F., 164.
Pell, A. J., 52.
Phillips, H. B., 279.
Pitcher, A. D., 443, 499, 499.
Putnam, T. M., 517.
Ranum, A., 52, 279.
Reaves, S. W., 219.
Reddick, H. W., 164.
Richardson, R. G. D., 388, 499.
Robinson, L. B., 52, 388.
Roever, W. H., 219.
Root, R. E., 442.
Rowe, J. E., 52, 278, 498.
Safford, F. H., 52.
Sanderson, M. L., 442.
Schottenfels, I. M., 279, 443.
Schweitzer, A. R., 444, 444.
Sharpe, F. R., 52, 52, 52, 278.
Shaw, J. B., 279, 279, 443.
Sheffer, H. M., 278, 498.
Silverman, L. L., 280.
Slepian, J., 498.
Slobin, H. L., 443.
Smail, L. L., 280, 498.
Snyder, V., 280.
Stager, H. W., 517.
Sullivan, C. T., 499.
Taylor, E. H., 164.
Vandiver, H. S., 499.
Veblen, O., 52, 388.
Volterra, V., 164.
Westlund, J., 278.
White, H. S., 52.
Whitney, A. W., 174.
Wilczynski, E. J., 53, 279, 443.
Wilder, C. E., 388.
Williams, K. P., 443, 499.
Winger, R. M., 280.
Young, J. W., 499, 499.

Personal Notes:

Ackermann-Teubner, A., 39; Akers, O. P., 380; Albright, G. H., 154, 211; Alexander, J. W., II, 154; Allardice, R. E., 101; Allen, E. S., 378; Almansi, E., 40, 40; Andoyer, H., 379; Appell, P., 266; Arnold, H. D., 34; Ashton, C. H., 204; Atchison, C. S., 101.

Bachiller, S. P., 40; Baker, H. F., 430; Baker, R. P., 204; Bartel, K., 211; Beckett, C. H., 101; Bell, E. T., 42; Bernays, P., 379; Birkhoff, G. D., 540; Bliss, G. A., 387, 484; Bôcher, M., 380; Boggio, T., 154; Bolza, O., 40, 325; Boothroyd, S. L., 100; Borel, E., 41; Boutroux, P., 323, 488; Brasefield, S. E., 34, 541; Brenke, W. C., 204; Brown, S. J., 100; Bussey, W. H., 204; Byerly, W. E., 211.

Cajori, F., 204; Callendar, H. L., 95; Cantor, G., 324; Carathéodory, C., 379; Carmichael, R. D., 42, 204; Carruth, W. M., 101; Cartan, E., 153; Chatelet, H., 433; Chazy, J., 323; Chittenden, E. W., 34; Cisotti, U., 488; Claude, A., 485; Combebiac, G., 155; Coolidge, J. L., 486, 540; Cramblet, H. W., 541; Crelier, L., 267.

Daniell, P. J., 41; Daniels, A. L., 34; Darwin, G. H., 267; Debye, P., 323, 485; Denton, W. W., 34; Dickson, L. E., 94, 429, 489; Dines, L. L., 34; Dingler, H., 211; Disteli, M., 379; Doehlemann, K., 41; Dohmen, F. J., 434; Douglass, H. M., 154; Dresden, A., 101; Duhem, P., 40; Duval, E. P. R., 42.

Eddington, A. S., 488; Eisenhart, L. P., 488; Engel, F., 265, 324, 433; Enriques, F., 266, 324; Epsteen, S., 541; Evans, G. C., 41, 378; Evans, W. D., 41.

Fehr, H., 95, 205, 430; Fiedler, W., 267; Field, F., 100; Fields, J. C., 265, 434; Finkel, B. F., 204; Finsterwalder, S., 41; Fischer, C. A., 34; Focke, T. M., 265; Forsyth, A. R., 325, 434; Fort, T., 35, 378; Fueter, R. v., 205, 324.

Gariel, C., 149; Garnier, R., 323; Gaylord, H. D., 434; Gérardin, A., 539; Gilman, R. E., 154; Glover, J. W., 541; Gordan, P., 325; Gouwens, C., 101; Graustein, W. C., 325, 378; Greenhill, G., 95; Gronwall, T. H., 434; Guichard, C., 154, 433, 538.

Hadamard, J., 266; Hamel, G., 266; Hartogs, F., 211; Haskell, M. W., 42; Hausdorff, F., 540; Hecke, E., 211; Hedrick, E. R., 204; Heffter, L., 40; Heiberg, J. L., 153; Hellebrand, E., 434; Hennel, C. B., 35; Henrici, O., 40; Hermite, C., 153; Heun, K., 379; Heywood, H. B., 41, 434; Hilbert, D., 324; Hilton, H., 154; Hitchcock, R. R., 101; Hoborski, A., 379; Hobson, E. W., 324; Hodgson, J. E., 154; Horn, J., 324; Hudson, H. P., 434; Humbert, G., 153; Huntington, E. V., 540; Hurwitz, W. A., 208.

Jacobsthal, E., 540; Jung, H., 433.

Kaderavek, F., 211; Karman, T. v., 266, 434; Kasner, E., 387, 484; Kellogg, O. D., 42; Killam, S. D., 101, 541; Kinkelin, H., 325; Klein, F., 95, 211, 379; Kobold, H., 323; Kohn, G., 40; Korn, A., 538; Kotter, F., 101; Kounowski, J., 40; Kuschke, C. G. P., 35.

Landsberg, G., 211; Lang, C. L., 41; Larmor, J., 489; Lauricella, G., 325; Lebesgue, H., 323; Lebon, E., 150; Legras, G., 42; Lemoine, E., 39; Leuschner, A. O., 489; Levi, E. E., 154; Lévy, L., 101; Lipke, J., 35; Lodge, O., 430; Long, M., 41; Lorentz, H. A., 153, 323, 485; Loria, G., 100; Love, A. E. H., 205; Lovett, E. O., 41.

MacMahon, J., 155; Maddrill, J. E., 154; Maillet, E., 39; Majcen, G., 434; Manchester, J. E., 380; Marcolongo, R., 100; Merlani, A., 486; Mikesh, J. S., 434; Miller, G. A., 204; Milne, W. E., 434; Mittag-Leffler, G., 266; Modigliani, G., 100; Mohrmann, H., 380, 433; Moody, R. H., 379; Morgan, F. M., 35; Mourgnot, M., 539; Moulton, F. R., 265; Mühll, K. v. der, 42; Murphy, L. W., 100.

Nernst, W., 323, 485; Netto, E., 324, 433; Nicholson, J. W., 154; Nörlund, N. E., 380.

Osgood, W. F., 94, 429.

Paine, G. P., 100; Painlevé, P., 154, 266; Palmer, E. S., 267; Paul, H. M., 100; Pfeiffer, R., 153; Picard, E., 40, 266, 324, 540; Pickering, E. C., 149; Pieri, M., 435; Planck, M., 323, 485; Poincaré, H., 43, 153, 266, 323; Poncelet, J. V., 153; Pöschl, T., 434; Prentiss, R. W., 435; Ptasickij, J. L., 42; Power, M., 41.

Rayleigh, Lord, 540; Rebière, 539; Reissner, H., 324; Richards, E. L., 42; Richardson, R. G. D., 42; Robbins, R. B., 434; Roe, H. B., 154; Root, R. E., 35; Rosenblatt, A., 488; Rosenthal, A., 211; Ross, A. D., 324; Rothe, R., 324; Ruhm, G., 267; Runge, C., 153; Russell, W. P., 101; Rychlik, M., 40.

Schäfer, E. A., 95; Schimmack, R., 267; Schlesinger, F., 265; Schoute, P. H., 489; Schur, F., 379, 486; Schur, I., 540; Schwarz, H. A., 266; Scorza, G., 41; Sheldon, E. W., 42; Siceloff, L. P., 35; Signorini, A., 434; Skeat, W. W., 155; Skiles, W. V., 100; Slaught, H. E., 204; Smith, D. E., 95, 322; Smith, E. S., 154; Smith, H. E., 100; Smith, P. F., 434; Smith, W. M., 35, 42; Smoluchowski, M. v., 323, 485; Somigliana, C., 154; Sommerfeld, A., 323, 485; Staeckel, P., 324; Stone, R. B., 42; Study, E., 379; Stuyvaert, M., 379; Sullivan, C. T., 35, 489.

Tait, P. G., 379; Tanner, J. H., 325; Toeplitz, O., 324; Tracey, J. I., 35; Treutlein, P., 155.

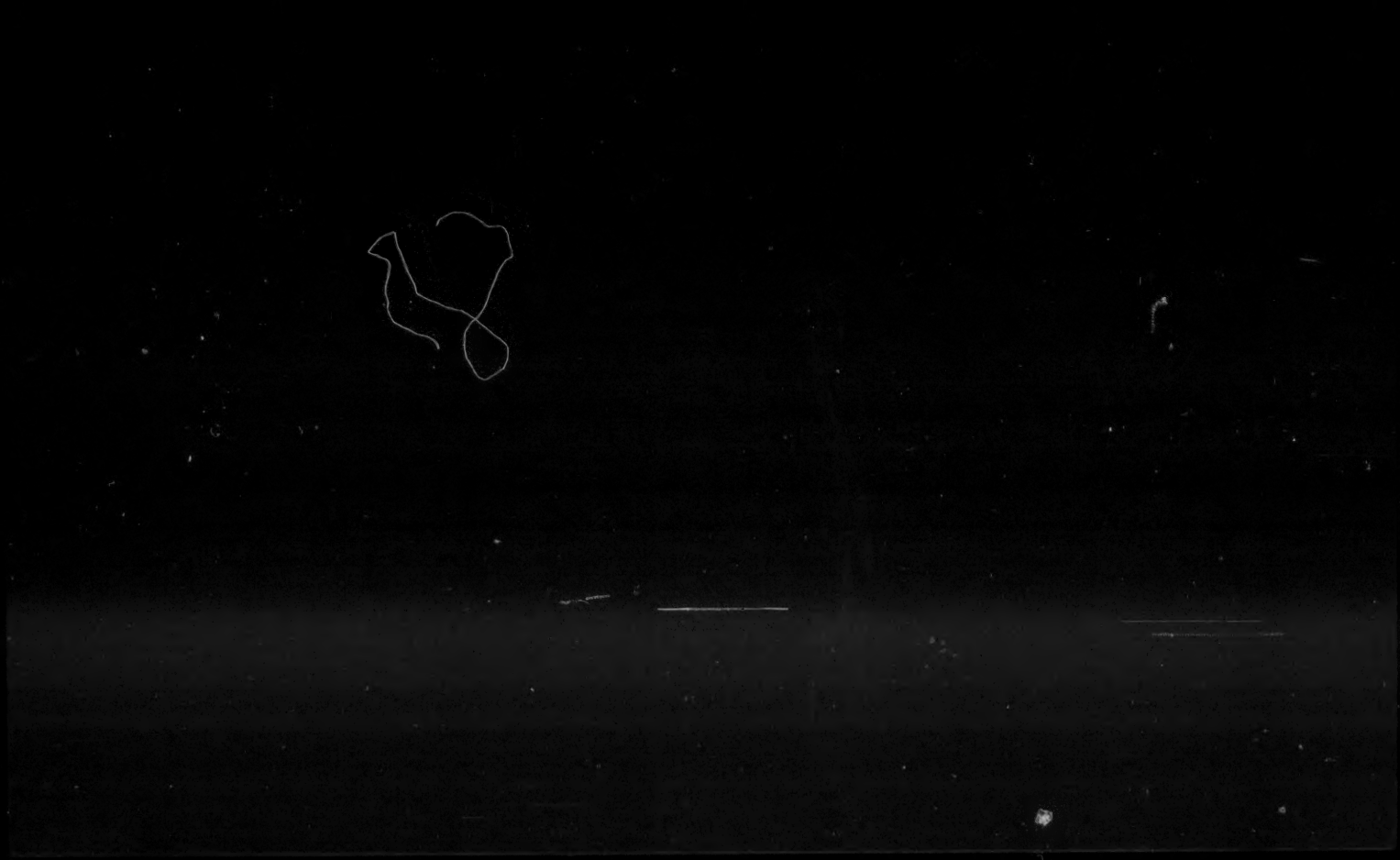
Urner, S. A., 101.

Van Vleck, E. B., 149; Van Vleck, J. M., 155; Vaughn, W. J., 267; Vessiot, E., 154; Volterra, V., 41.

Wahlin, G. E., 101; Warren, L. A. H., 267, 380; Weber, H., 541; Westfall, J. V. E., 211; Whitehead, A. N., 41, 153; Whitford, E. E., 35; Whittaker, E. T., 41, 323; Wilder, C. E., 434; Willard, H. R., 35; Williams, K. P., 155; Wilson, A. H., 35; Wilson, E. B., 265; Wilson, L. T., 434; Wilson, W. A., 101; Winger, R. M., 35; Wirtz, C. W., 323; Woods, B. M., 35.

Prizes:

Ackermann-Teubner, 39; Beneke, 210; Belgian Academy, 538; Berlin Academy, 39; Bologna Academy, 486; Copley Medal, 210; Henrici Medal, 40; Italian Society of Sciences, 40, 154; Lobachevsky, 379, 485; Paris Academy, 39, 153, 323, 430, 485, 538.

Universities:

Berlin, 97, 487.
Bologna, 37.
Bonn, 35, 377.
Breslau, 97.
Cambridge, 151.
Catania, 37.
Chicago, 376.
Columbia, 431, 431.
Cornell, 432.
Erlangen, 97.
Genoa, 37.
Giessen, 97.
Göttingen, 97, 377.
Greifswald, 98.
Halle, 35.
Harvard, 378, 432.
Indiana, 486, 486.
Jena, 98.
Johns Hopkins, 539.
Kiel, 98.
Leipzig, 36, 488.
Munich, 36, 378.
Münster, 99.
Naples, 37.
Oxford, 152.
Padua, 38.
Palermo, 38.
Paris, 99, 153, 266, 433.
Pavia, 38.
Pennsylvania, 376.
Pisa, 38.
Princeton, 540.
Rome, 38.
Strassburg, 36, 488.
Tübingen, 99.
Turin, 39.
Yale, 487.